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# Nonideal Effects in SC circuits

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# Components and Nonidealities

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- **Switches:**

Nonzero “on”-resistance

Clock feedthrough / charge injection

Junction leakage, capacitance

Noise

- **Capacitors:**

Capacitance errors

Voltage and temperature dependence

Random and systematic variations

Leakage

- **Op-amps:**

DC offset voltage

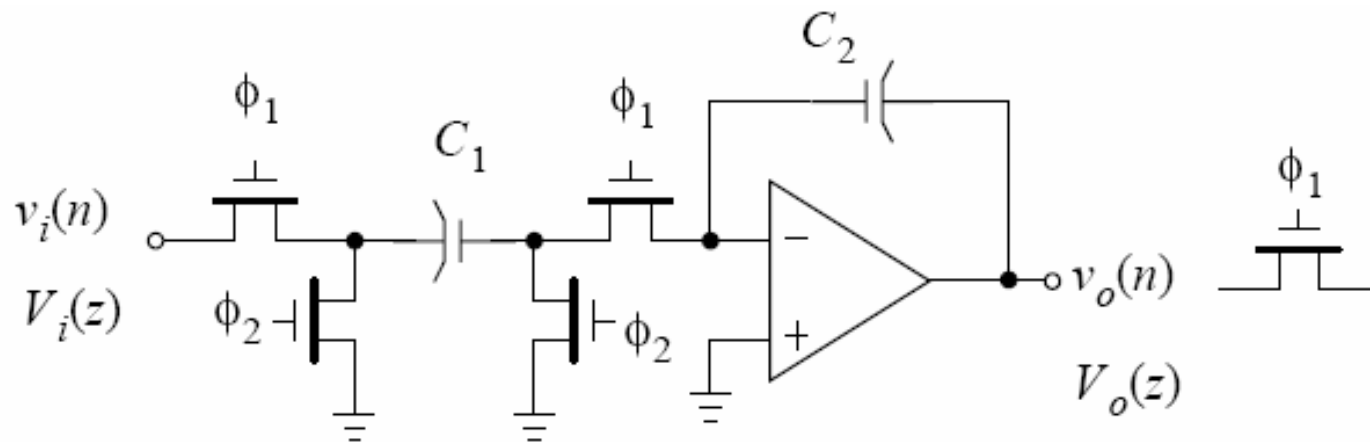
Finite dc gain

Finite bandwidth

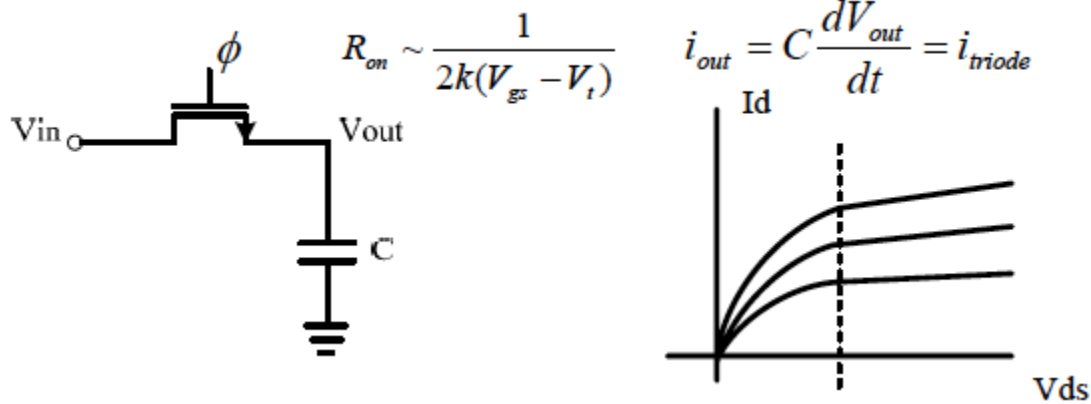
Nonzero output impedance

Noise

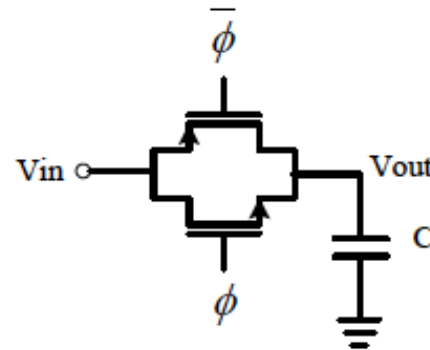
# Switched-Capacitor Integrator



# Nonzero Switch “On”-Resistance

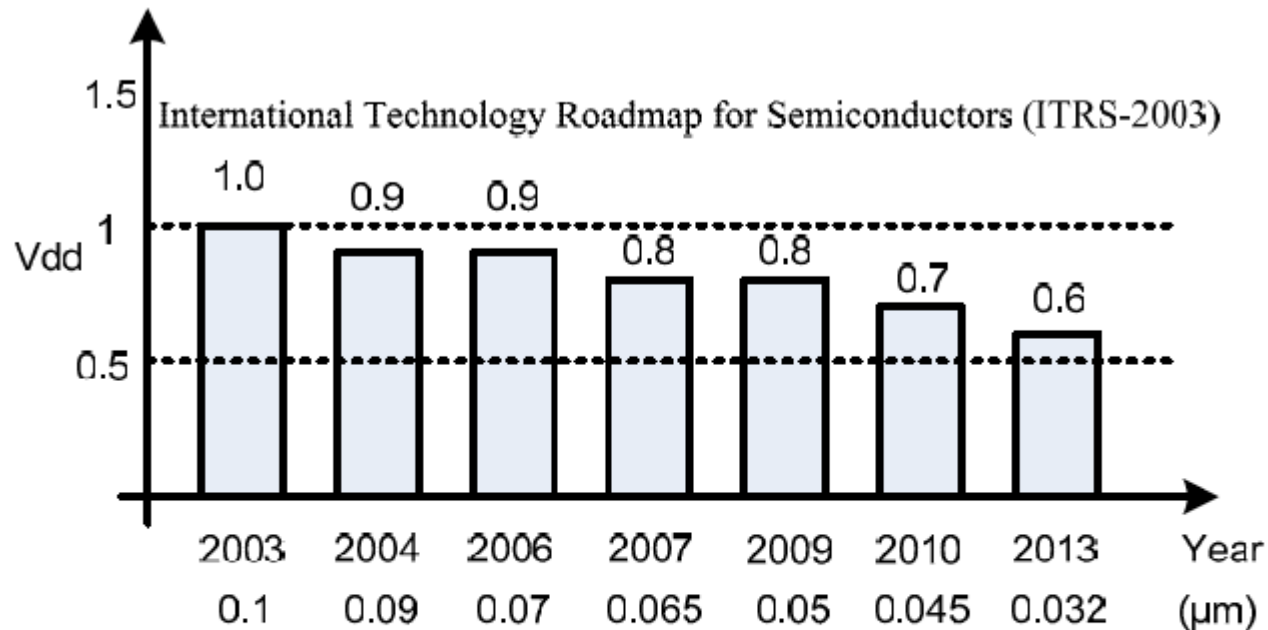


C is charged exponentially. Time constant must be sufficiently low. Body effect must be included!



To lower on-resistance and clock feedthrough: CMOS gate;  $W_p, L_p = W_n, L_n$ . For settling to within 0.1%,  $T_{settling} > 7R_{on}C$  (worst-case  $R_{on}$ ).

# Digital CMOS Scaling Roadmap



Want analog circuit to operate at low supply voltage

# Predicted Dimensions

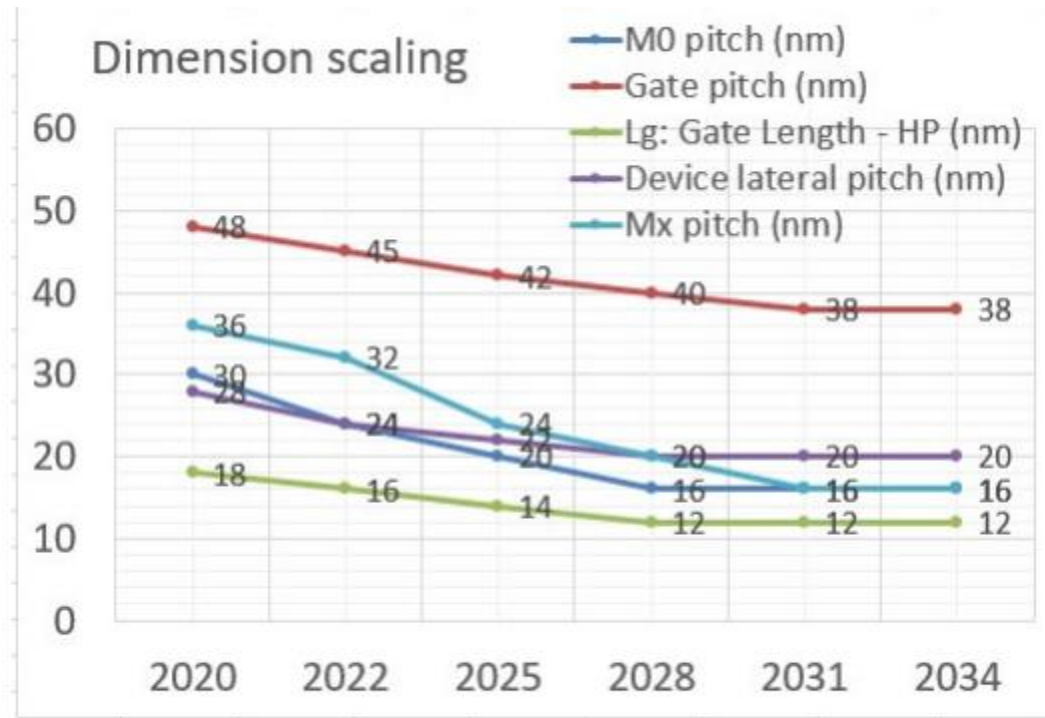
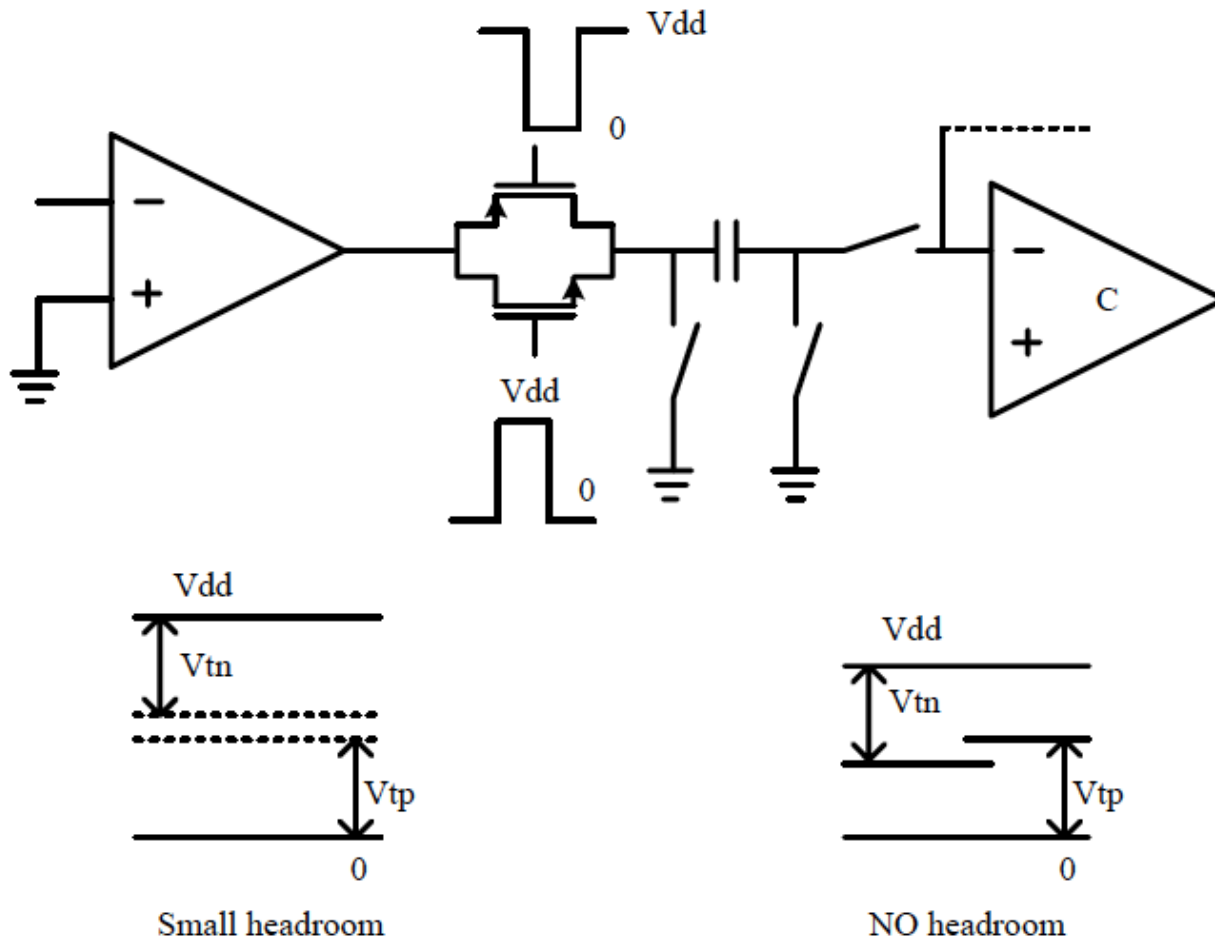


Figure MM-2

Projected scaling of key ground rules

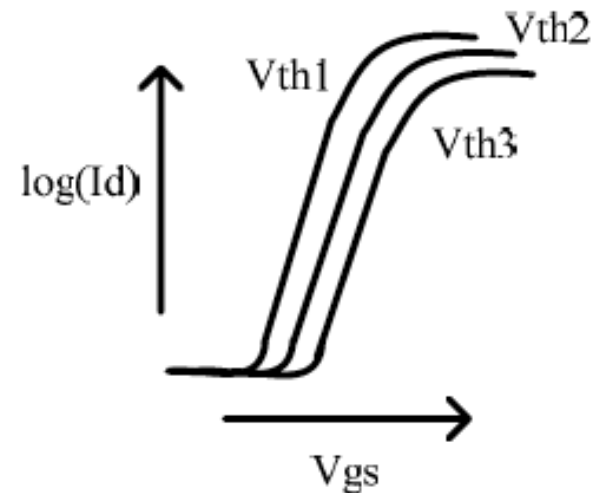
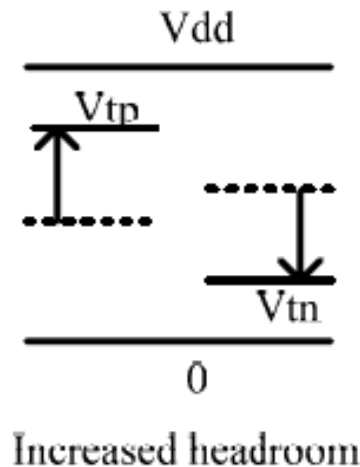
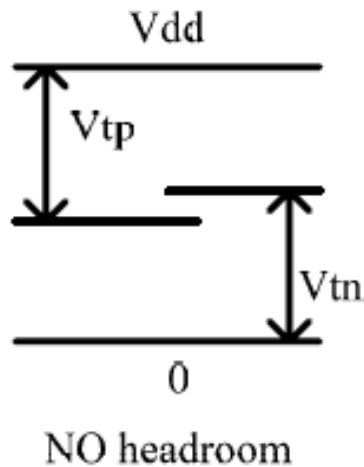
# Floating Switch Problem in Low-Voltage



• *FORBIDDEN REGIONS FOR SIGNAL*

# Using Low-Threshold Transistors

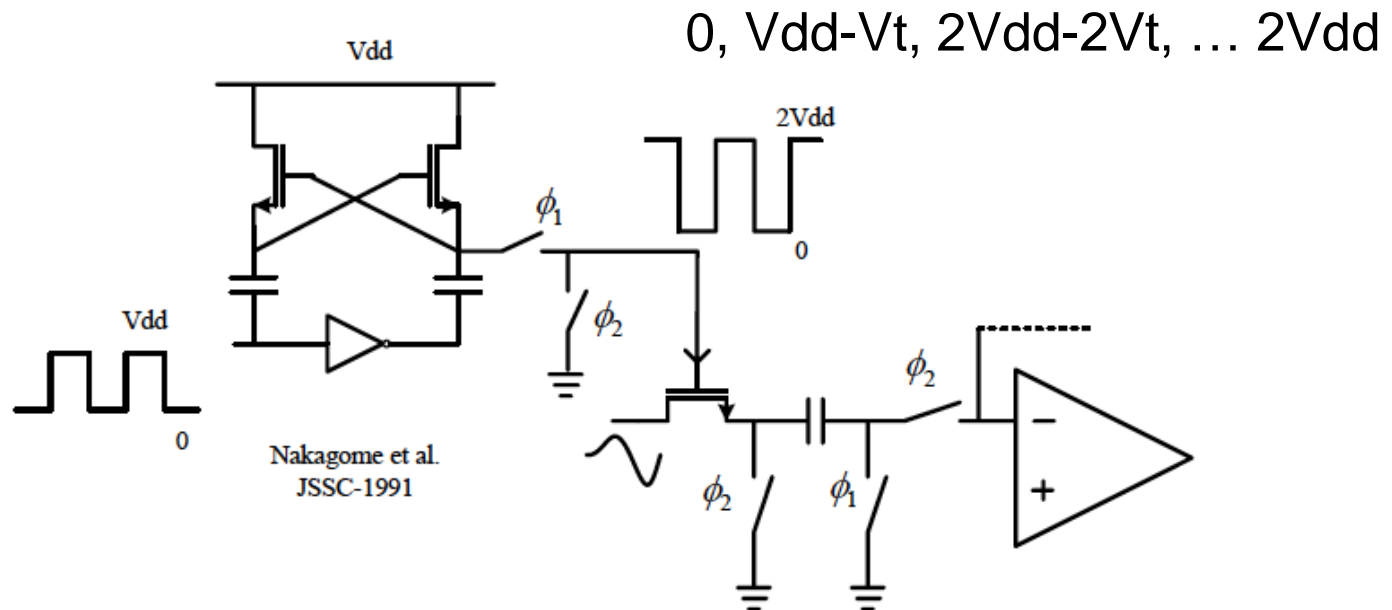
- Precise control over process and temperature difficult
- Switch leakage worsens as threshold voltage is lowered (i.e. hard to turn off)





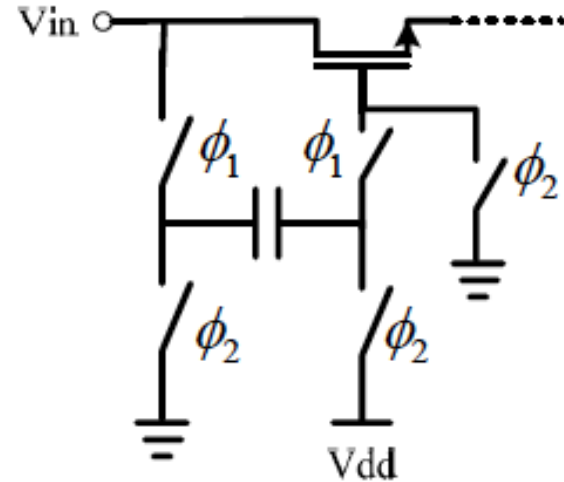
# Using Clock Voltage Booster

- Boosted clock voltage (e.g.  $0 \rightarrow 2V_{dd}$ ) is used to sufficiently overdrive the NMOS floating switch – useful in systems with low external power supply voltage and fabricated in high-voltage CMOS process  
Battery, harvested energy may be used!
  - Voltage limitation is violated in low-voltage CMOS



# Using Bootstrapped Clock

Abo et al., JSSC-1999  
Steensgaard, ISCAS-1999  
Singer et al., ISSCC-2000  
Dessouky et al., JSSC-2001  
(fast) Hernes et al., ISSCC-2004

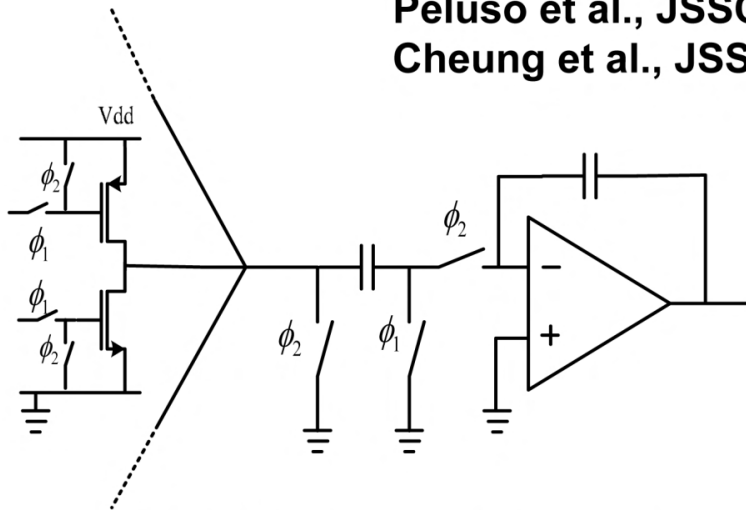


- Principle: pre-sample  $V_{dd}$  before placing it across  $V_{gs}$  (various low-voltage issues complicate implementation)
- Input sampling such as this can be used for low-voltage CMOS or for high-linearity sampling
- No fundamental or topological limitation on higher input signal frequency vs. sampling frequency
- *BUT BREAKDOWN, ADDED SWITCH TURN-ON PROBLEM.*

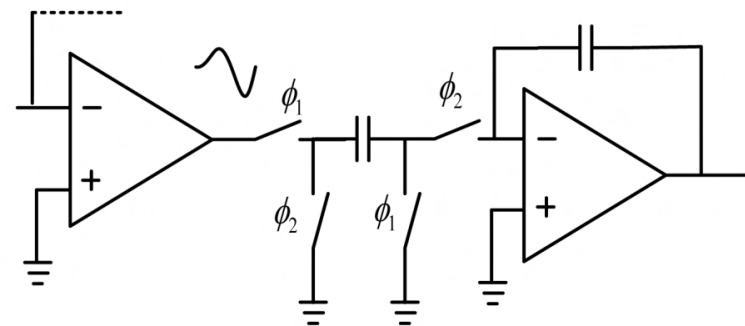
# Switched-Opamp Technique

- Floating switch is eliminated
- Opamp output tri-stated and pulled to ground during reset  $\phi_2$   
→ Slow transient response as opamp is turned back on during  $\phi_1$

Crols et al., JSSC-1994(1.5-V)  
Peluso et al., JSSC-1997(1.5-V)  
Baschiroto et al., JSSC-1997(1-V)  
Peluso et al., JSSC-1998(1-V)  
Cheung et al., JSSC-2002 (1-V)



Switched-opamp integrator



Conventional integrator

# Switched-Opamp Example

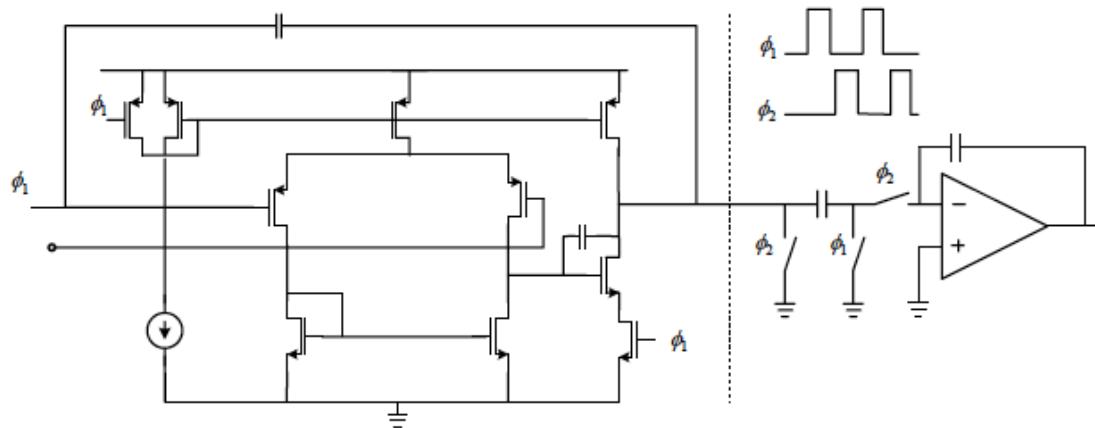
Crols et al., JSSC-1994

Peluso et al., JSSC-1997

The entire opamp is turned off during  $\phi_2$ .

Demonstrated 1.5-V operation ( $\Delta\Sigma$ ) with  $V_{tn}=|V_{tp}|=0.9V$ .

115kHz at -60dB THD & 500kHz at -72dB THD.



→ **Baschiroto JSSC-1997: 1-V operation**

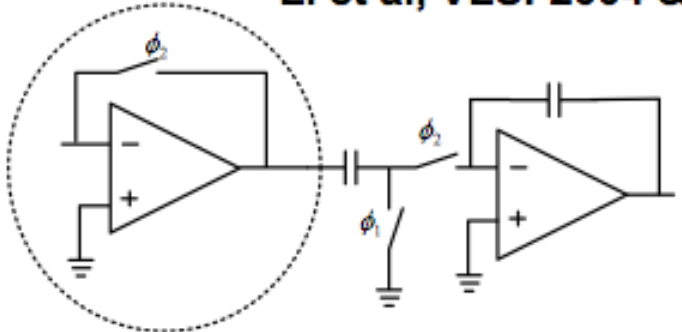
→ **Peluso JSSC-1998: 0.9-V operation**

**1.8MHz at -40dB THD**

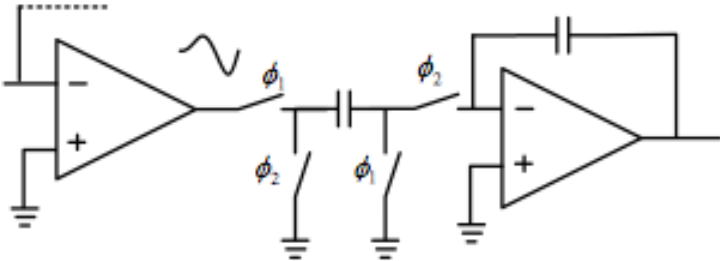
**Cheung et al., JSSC-2002(1-V)**

# Opamp-Reset = Unity-Gain Configuration

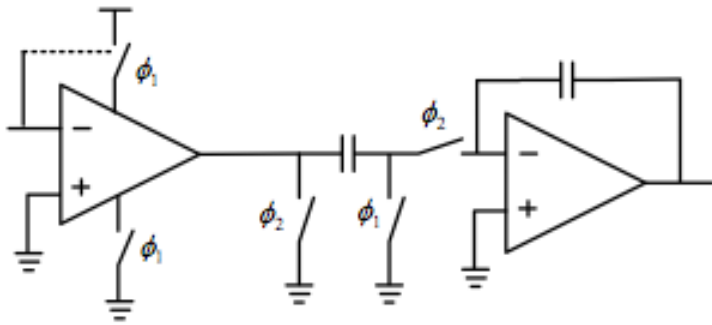
Bidari et al., ISCAS-1999  
Keskin et al, ESSCIRC-2001 & JSSC-2002  
Chang et al, CICC-2002 & JSSC-2003  
Chang et al, VLSI-2003 & TCAS1-2005  
Li et al, VLSI-2004 & JSSC-2005



- High speed operation
- Free of reliability issues



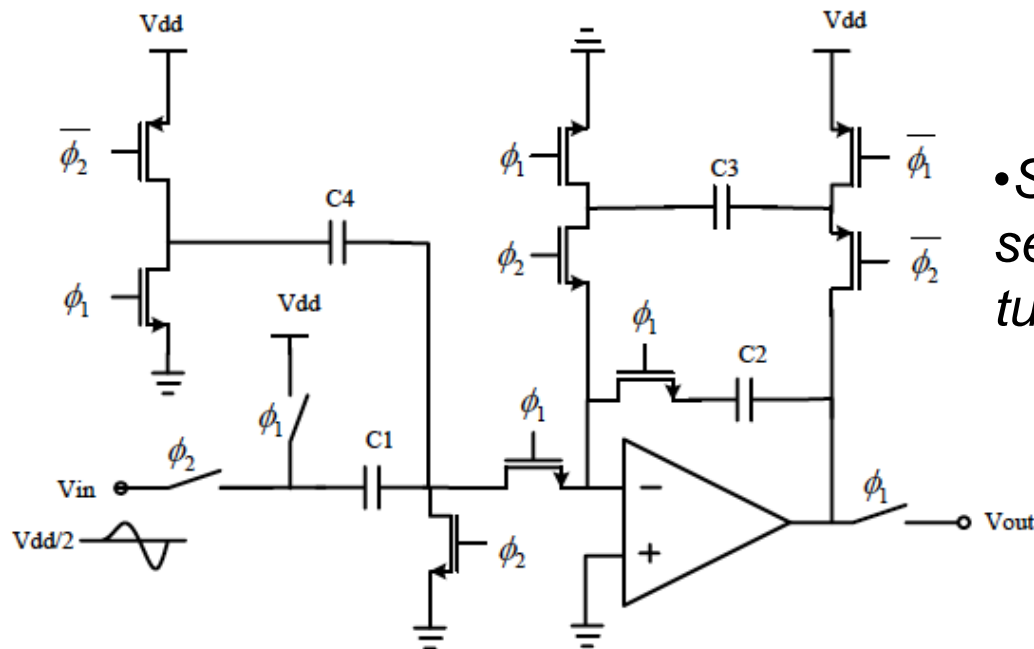
Conventional integrator



Switched-opamp integrator

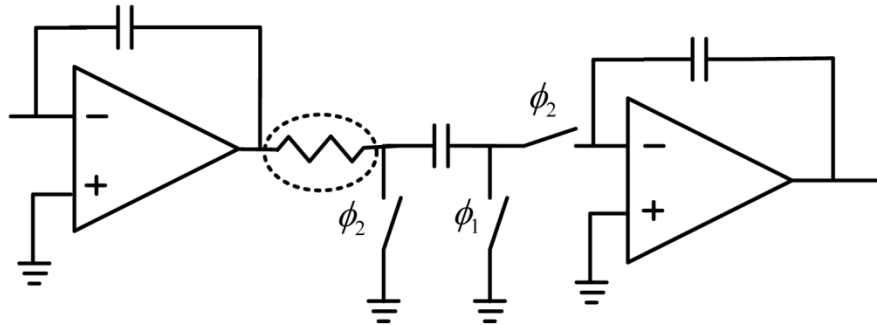
# Floating Reference Avoids Fwd Bias

- C3 is precharged during  $\phi_1$
  - C3 (floating reference) in feedback during  $\phi_2$
  - DC offset circuit ( $C4=C1/2$ ) compensates for Vdd reset of C1
- effective virtual ground =  $V_{dd}/2$



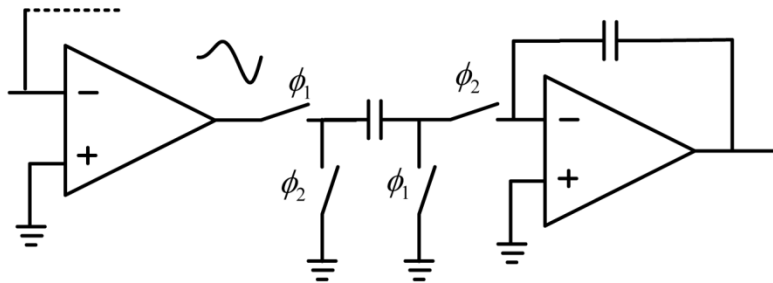
• *Simpler: 2 switches in series with  $C2$ , grounded turned off first.*

# Switched-RC = Resistor Isolation

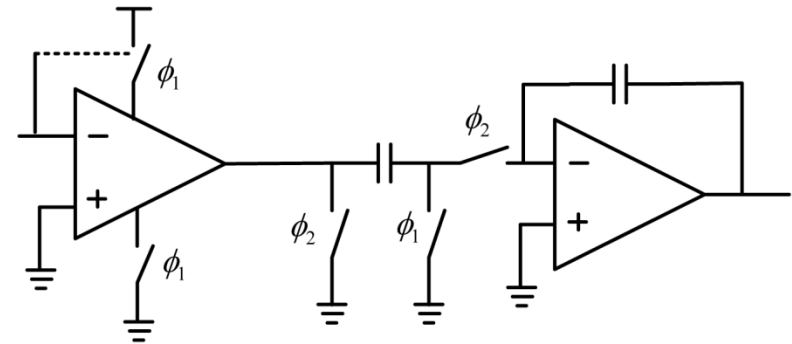


- No floating switch
- Highly linear sampling
- Free of reliability issues

• *R must be chosen properly.*



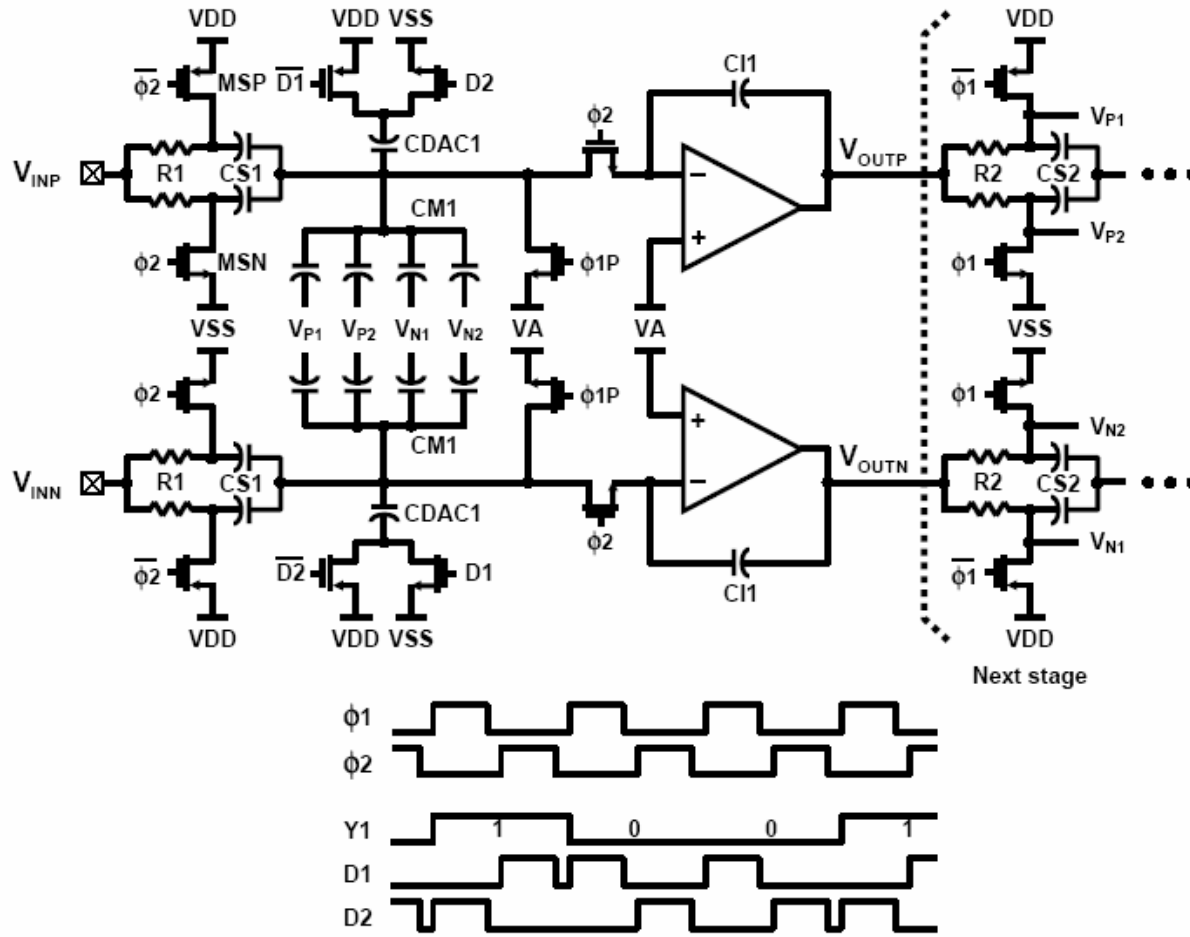
Conventional integrator



Switched-opamp integrator

# Switched-RC = Resistor Isolation

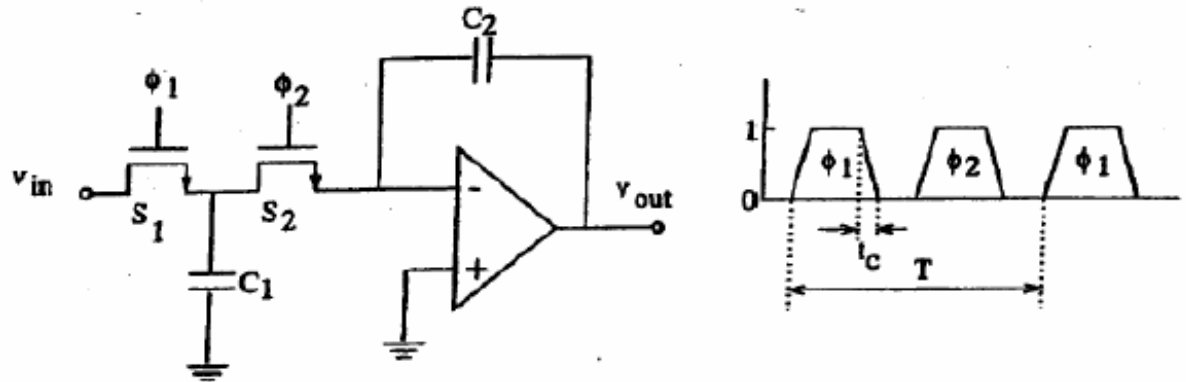
Ahn et al., ISSCC-2005 Paper 9.1



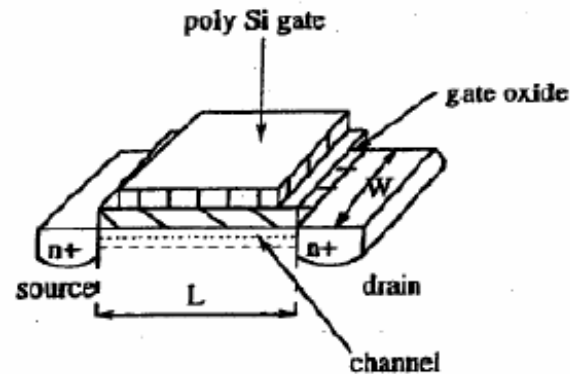


# Charge Injection (1)

- Simple SC integrator



S<sub>1</sub> structure



# Charge Injection (1) (Cont'd)

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- The lateral field is  $v/L$ , the drift velocity is  $\mu v/L$ . Therefore, the current is

$$i = q_{ch} \cdot \mu \cdot v / L^2$$

- The on-resistance is

$$R_{on} = v / i = L^2 / (q_{ch} \cdot \mu)$$

- and hence

$$R_{on} \cdot q_{ch} = L^2 / \mu$$

holds.

# Charge Injection (2)

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From device physics,

$$q_{ch} = -W \cdot L \cdot C_{ox} \cdot (V_{dd} - v_{in} - V_{tn})$$

Unless S1 is in a well, connected to its source,  $V_{tn}$  depends on  $V_{in}$ , so  $q_{ch}$  is a mildly nonlinear function of  $V_{in}$ .

When S1 cuts off, part of  $q_{ch}(q_s)$  enters  $C_1$  and introduces noise, nonlinearly, gain and offset error.

To reduce  $q_s$ , choose  $L$  small, but and  $R_{on}$  large. However, for 0.1% settling

$$R_{on} \cdot C_1 < T/14 = 1/(14f_c)$$

Hence

$$q_{ch,\min} = L^2 / (R_{on,\max} \cdot \mu) = 14 \cdot L^2 \cdot f_c \cdot C_1 / \mu$$

and

$$v_{err,\min} = d \cdot q_{ch,\min} / C_1 = 14 \cdot d \cdot L^2 \cdot f_c / \mu$$

Pedestal voltage

where  $d = q_s / q_{ch}$

# Clock Feedthrough

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Capacitive coupling of clock signal via overlap  $C_{ov}$  between gate and source. The resulting charge error is

$$q_{ov} = -V_{dd} \cdot C_{ov} \cdot C_1 / (C_1 + C_{ov})$$

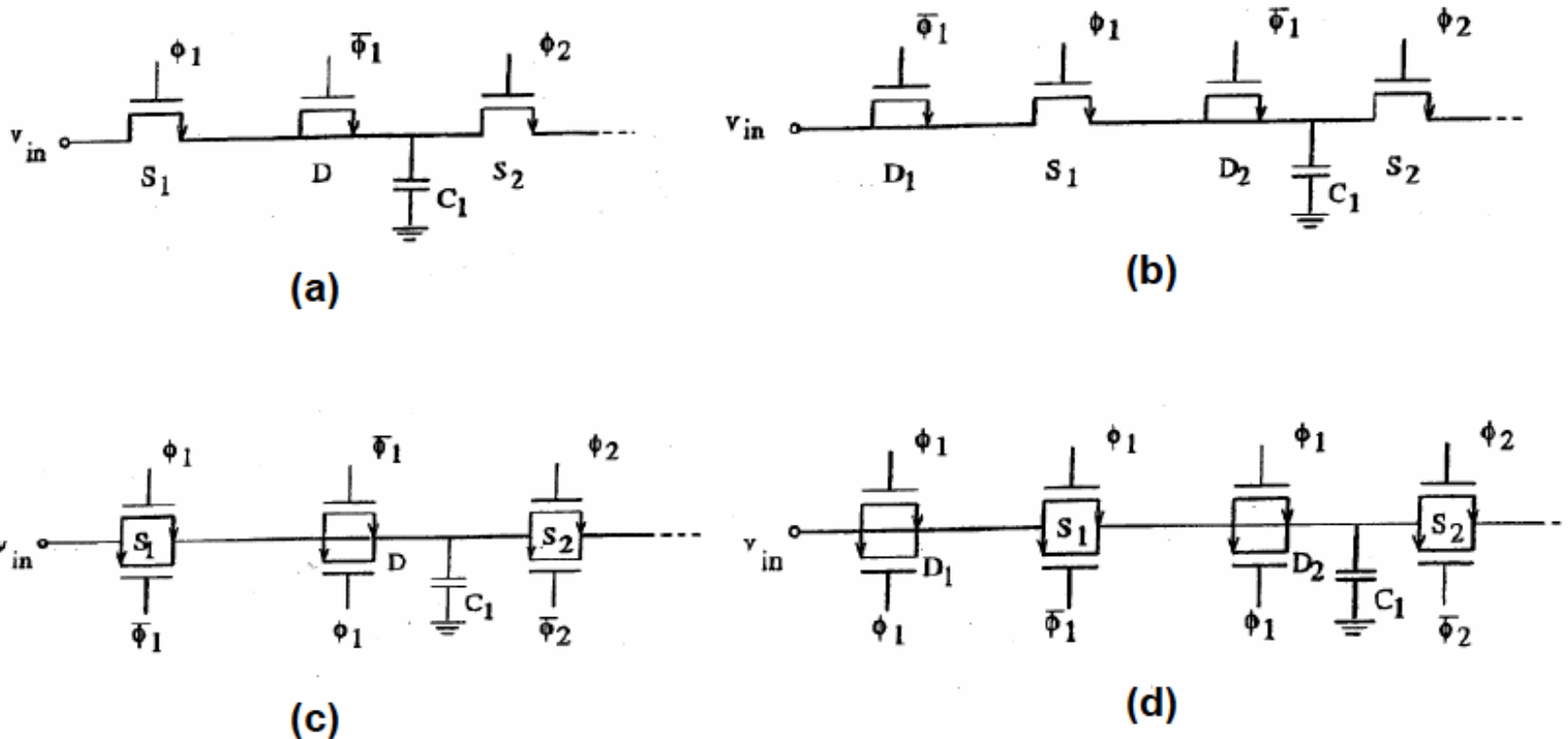
It adds to  $q_s$ . Usually,  $|q_{ov}| \ll |q_s|$

Linear error .

Same for  $S_1$  and  $S_2$ .

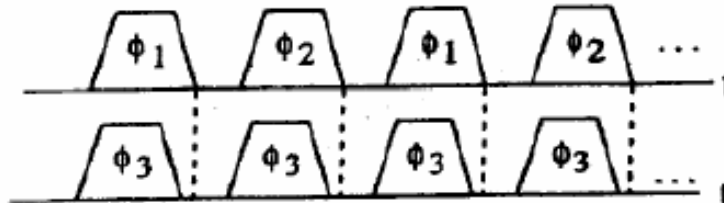
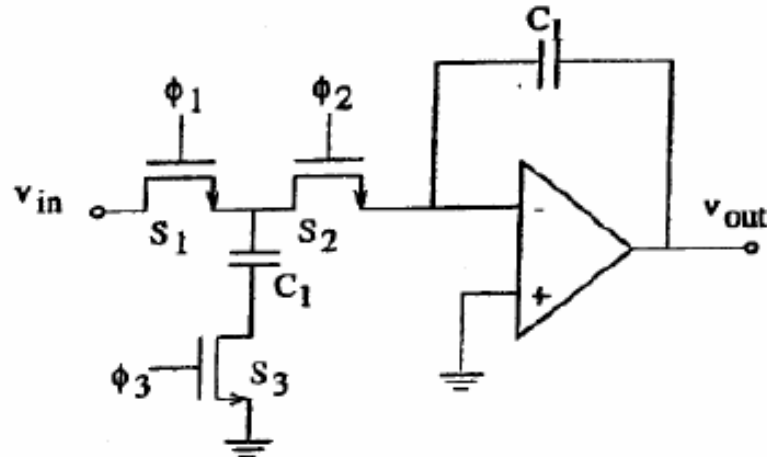
# Methods for Reducing Charge Injection

- Transmission gates: cancellation if areas are matched. Poor for floating switches, somewhat better for fixed-voltage operation.
- Dummy devices: better for  $d \sim 0.5$ .

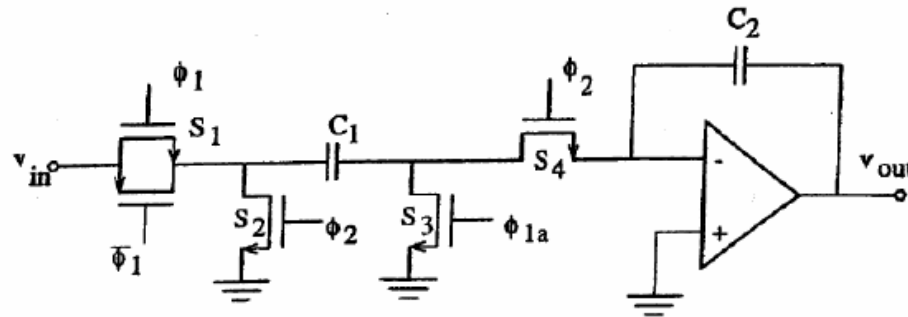


# Advanced-Cutoff Switches

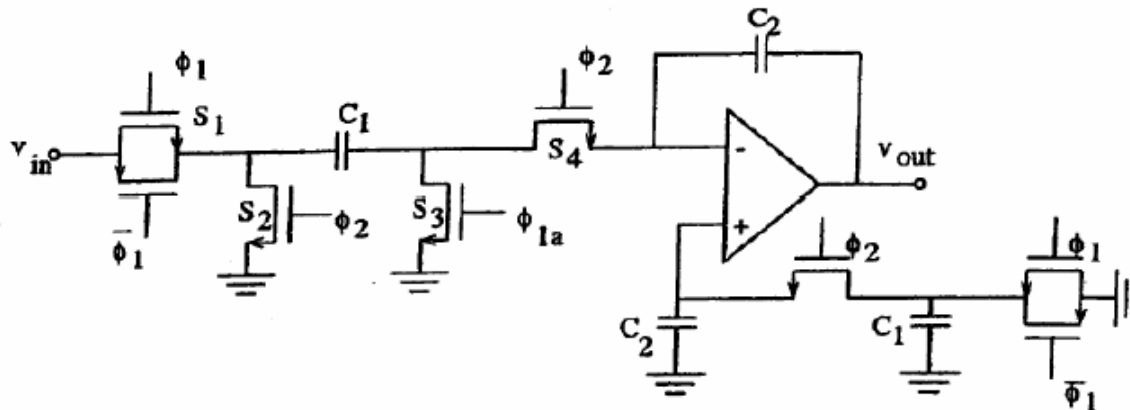
- Signal-dependent charge injection leads to nonlinear distortions; signal-independent one to fixed offset. Advanced-cutoff switches can reduce signal dependence.



# Advanced-Cutoff Switches (Cont'd)

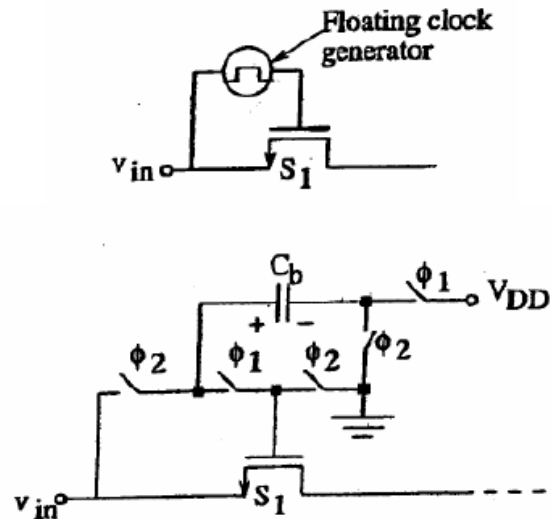


- Remaining charge injection is mostly common-mode in a differential stage.
- Suppressed by CMRR. In a single-ended circuit, it can be approximated by dummy branch:



# Floating Clock Generator

- To reduce signal dependence, reference the clock signal to  $v_{in}$ :

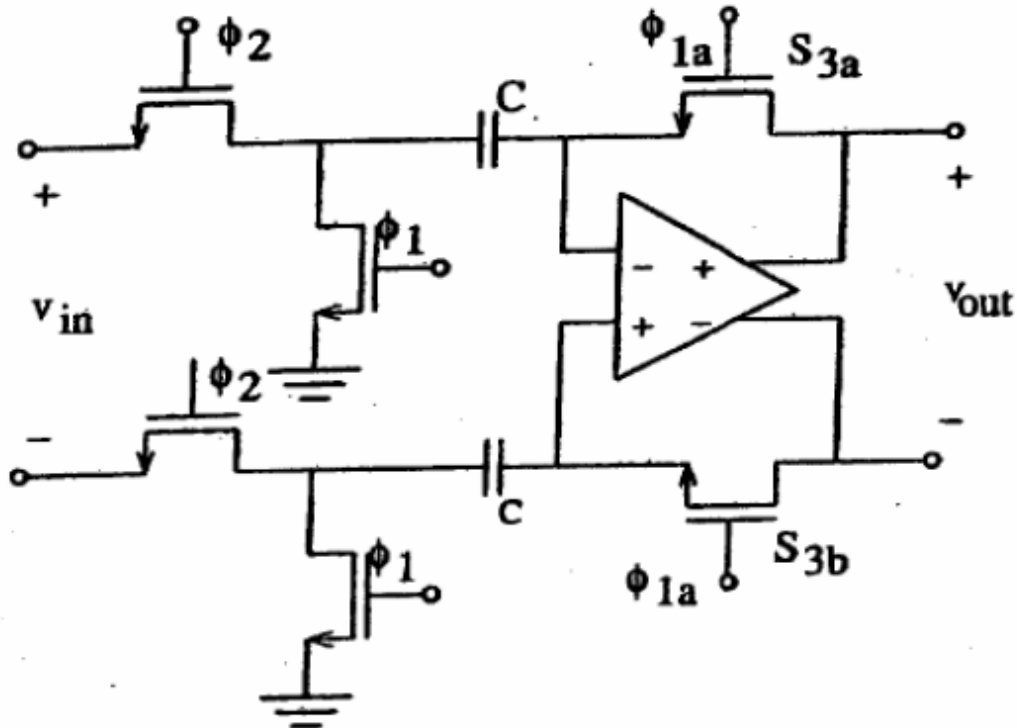


- This makes  $R_{on}$  also signal independent, so the settling is more linear. Clock feedthrough remains signal dependent, but it is a linear effect anyway. Better phasing : precharge  $C_b$  to  $V_{DD}$  during phase 2, connect to  $v_{in}$  during phase 1.

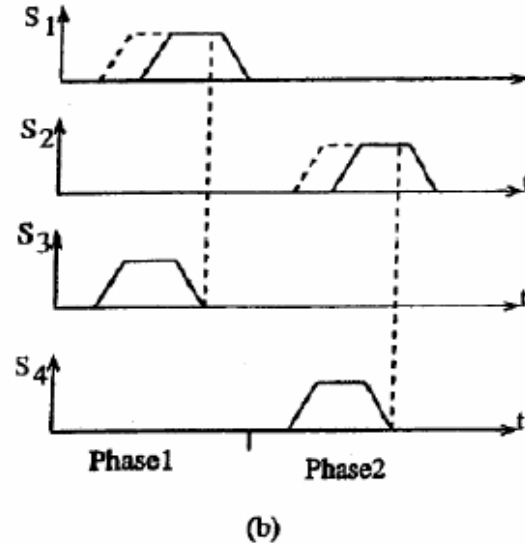
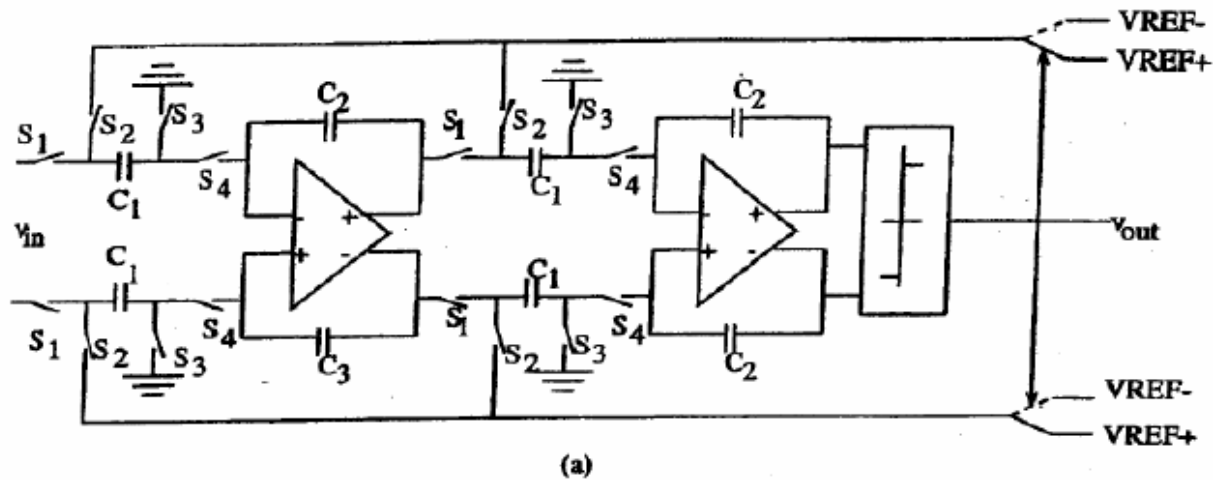


# Charge Injection in a Comparator

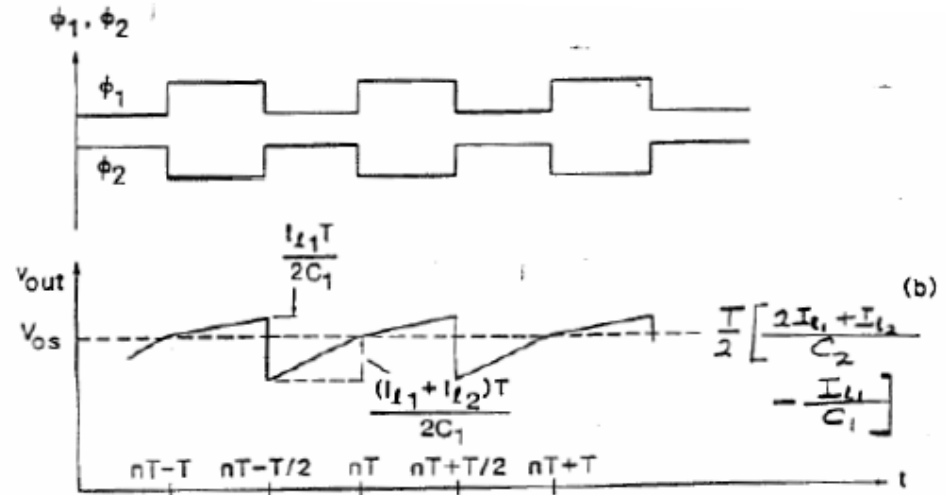
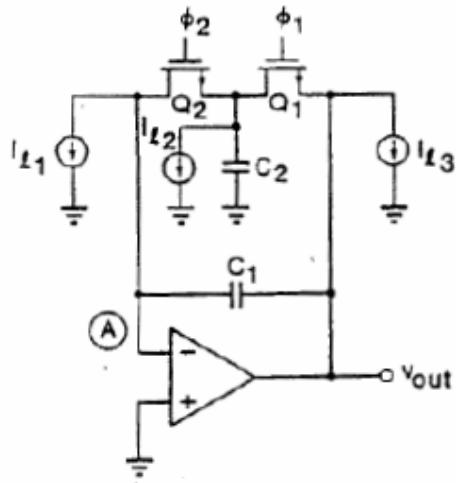
- Remains valid if input phases are interchanged.



# Delta-Sigma ADC



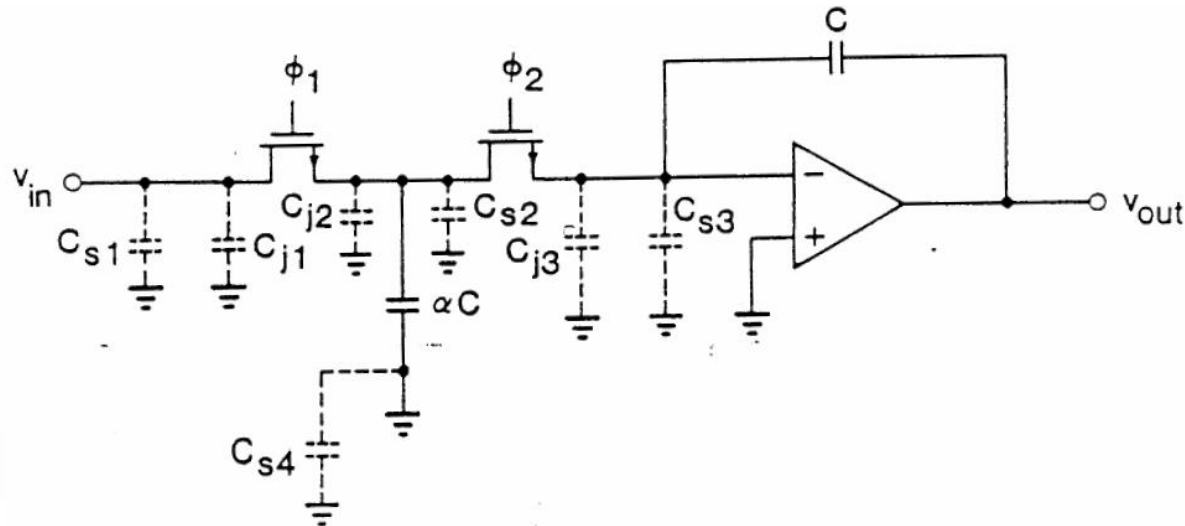
# Junction Leakage



- $I \sim 10 \text{ pA/mil}^2$  ,  $0.4\text{pA}/5\mu \times 5\mu$  but doubles for each  $10^\circ \text{ C}$ .
- $f_{\min} \sim 100\text{Hz}$  at  $20^\circ \text{ C}$ , but  $25\text{KHz}$  at  $100^\circ \text{ C}$ .
- Fully differential circuit and Martin compensation converts it to common-mode effect.

# Capacitances Inaccuracies

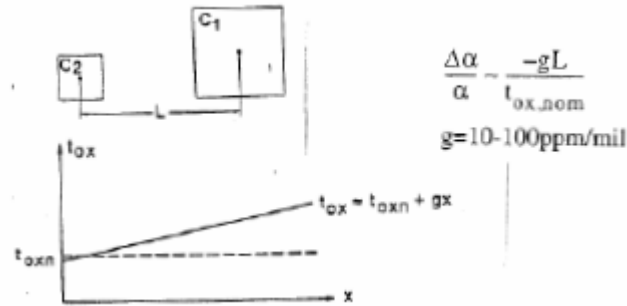
- Depends only on C ratios. Strays are often p-n junctions, leading to harmonic distortion also. For stray-sensitive integrator, all strays should be  $< 0.1\%$  of  $\alpha C$ .



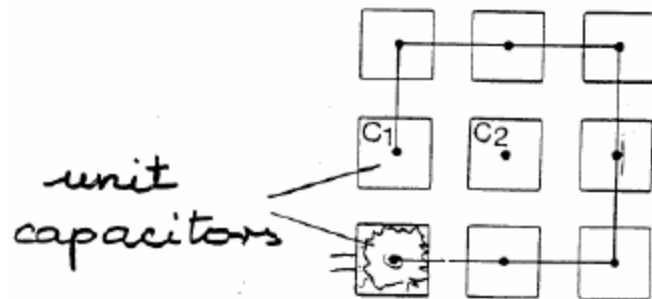
- $\Delta C$  can be systematic or random. Random effects (granularity, edge effects, etc.) cannot be compensated, but systematic ones can, by unit-capacitor/common-centroid construction of  $\alpha C$  and  $C$ .

# Capacitances Inaccuracies (Cont'd)

- Oxide gradient



- Common-centroid geometry



Compensated  $C_1/C_2$  against linear variations of  $C_{ox}$ , and edge related systematic errors (undercut, fringing)

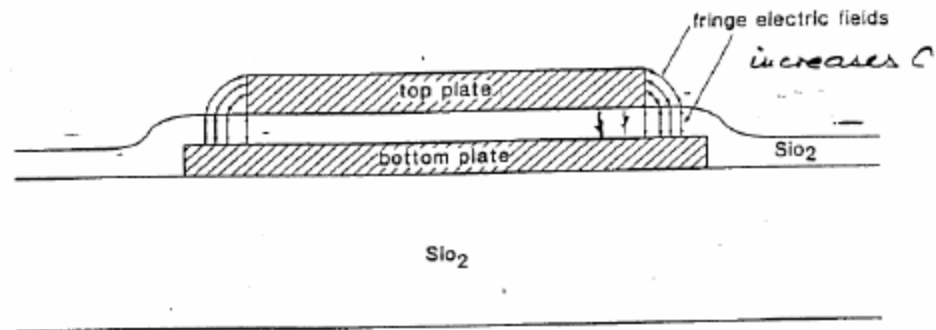
# Capacitances Inaccuracies (Cont'd)

- Voltage and temperature coefficients

$$\gamma_v^c = \frac{1}{C} \frac{\partial C}{\partial v} \quad \text{usually} \quad |\gamma_v^c| \sim 10 \text{ ppm/V}$$

$$\gamma_T^c = \frac{1}{C} \frac{\partial C}{\partial T} \quad \text{usually} \quad |\gamma_T^c| \sim 20 \text{ ppm/V}$$

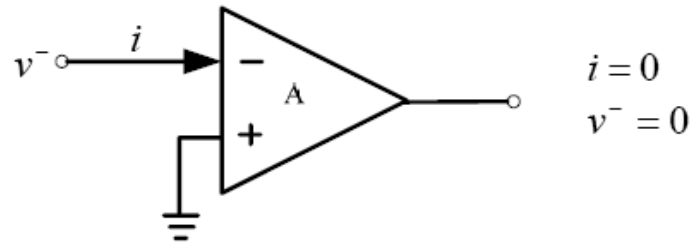
Smaller for ratios, especially for common-centroid layout:



Fringing, undercut: systematic edge effects. Reduced by commoncentroid geometry, since perimeter/area ratio is the same for C1 and C2,  $\Delta C \propto \text{perimeter}$  ,  $C \propto \text{area}$

# OPAMP Input Offset

- In most analog IC, the active element is the opamp. It is used to create a virtual ground (or virtual short circuit) at its input terminals:



- This makes lossless charge transfer possible. In fact, in a CMOS IC,  $i \approx 0$  but  $v \neq 0$  due to offset,  $1/f$  and thermal noise and finite opamp gain  $A$ . Typically,  $|v| = 5-10\text{mV}$ . This affects both the dc levels and the signal processing properties. The effect of  $v$  is even more significant in a low-voltage technology where the signal swing is reduced, and  $A$  may be low since cascoding may not be available.

# Techniques for Reducing the Effect of Imperfect Virtual Grounds

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- Autozeroing or Correlated Double Sampling Schemes:
  - Scheme A: Stores and subtracts  $v$  at the input or output of the opamp;
  - Scheme B: Refers all charge redistributions to a (constant)  $v$  instead of ground;
  - Scheme C: Predicts and subtracts  $v$ , or references charge manipulations to a predicted.
- Compensation using extra input: An added feedback loop generates an extra input to force the output to a reset value for zero input signal.
- Chopper stabilization: The signal is modulated to a “safe” (low-noise) frequency range, and demodulated after processing.
- Mixed-mode schemes: Establish a known analog input, use digital output for correction.

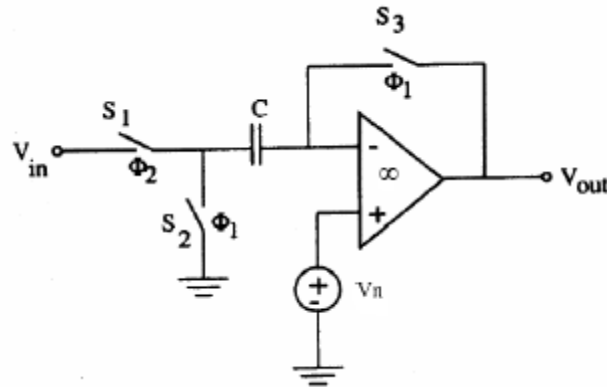


# Circuits Using Autozeroing

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- Comparators
- Amplifiers
- S/H, T/H, delay stages
- Data converters
- Integrators
- Filters
- Equalizers

# Simple Autozeroed Comparator



Nonidealities represented by added noise voltage:

$$v_n = v^- = V_{os} + v_{1/f} + v_{thermal} - \mu V_{out}; \quad \mu = 1/A_{opamp}$$

Input-referred noise at the end of interval:

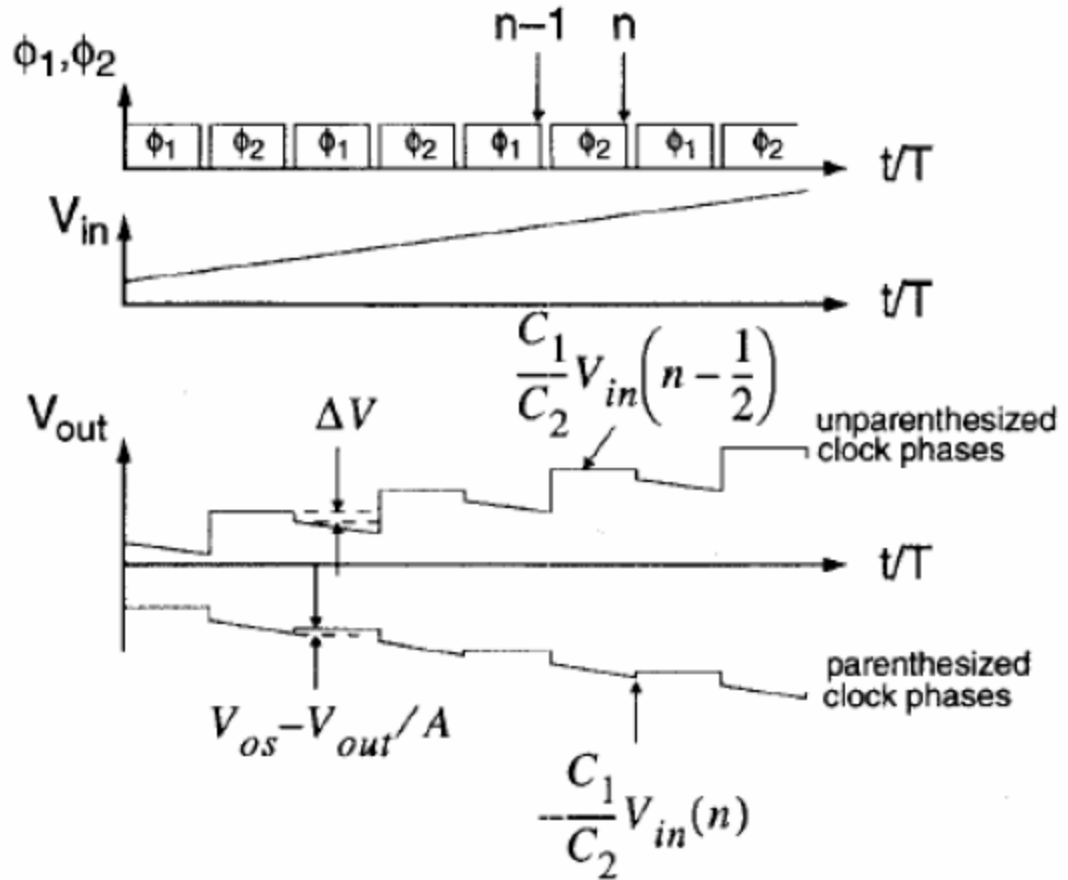
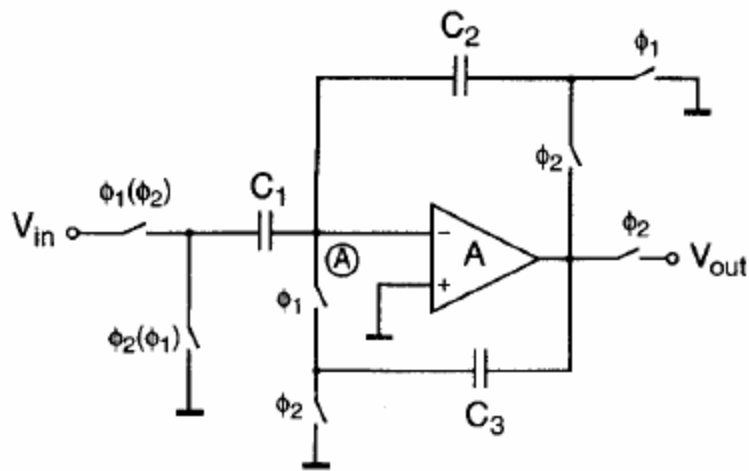
$$v_{n,in}(n) = v_n(n) - v_n(n-1/2);$$

Transfer function without folding:  $|H_N|^2 = 4 \sin^2(\omega T / 4)$

$V_{os}$ ,  $V_{1/f}$  and (for oversampled signals)  $\mu V_{out}$  may be reduced by  $H_N$ . Here,  $\mu V_{out}$  is not considered, since it is not important for a comparator.

# An Offset- and Finite-Gain-Compensated SC Amplifier

Haug et al., 1984 ISCAS



# Analysis of Compensated Gain Amplifier

Input-output relation for inverting operation:

$$v_{out}(n) = -C_1 / C_2 v_{in}(n) + (1 + C_1 / C_2)[v^-(n) - v^-(n-1/2)]$$

The S/H capacitor switches from 0 to

$$v^-(n) = V_{os} - \mu v_{out}(n-1/2) \quad \text{as } \phi_1 \rightarrow 1. \text{ Hence,}$$

$$v_{out}(n-1/2) - v_{out}(n-1) = V_{os} - \mu v_{out}(n-1/2), \text{ where } \mu = 1/A.$$

At low signal frequencies where  $v_{out}(n) \cong v_{out}(n-1)$ ,

the error term is only  $(1 + C_1 / C_2)\mu^2 v_{out}$ . The dc gain is

$$H(1) = \frac{-C_1 / C_2}{(1 + C_1 / C_2)\mu^2} \bullet + 1$$

The output step at reset is only  $A_v \cong V_{os} - \mu v_{out} - (C_1 / C_2)\Delta v_{in}$ , where the last term enters for noninverting operation only.  $A_v$  is usually 1~10 mV. The slewing required is minimal. The output offset is  $\mu(1 + C_1 / C_2)V_{os}$ .

Error in H(1): denom. should have + 1. Clock feedthrough generates some residual offset. Can be used as a compensated delay stage.

# Finite Opamp DC Gain Effect

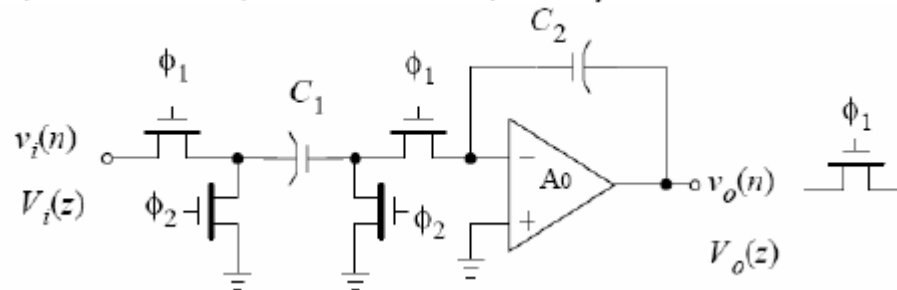
For  $A_0 \rightarrow \infty$ ,  $H_i(e^{j\omega T}) = \frac{-(C_1/C_2)e^{(j\omega T)/2}}{2 \sin((\omega T)/2)}$

For  $A_0 < \infty$ ,  $H(e^{j\omega T}) = (1 + m(\omega))e^{j\theta(\omega)}H_i(e^{j\omega T})$ .

Here, the relative gain error  $m \equiv -(1/A_0)(1 + C_1/(2C_2))$

the relative phase error  $\theta \equiv (C_1/C_2)/(A_0\omega T)$ .

(Martin, PhD thesis, U of Toronto, 1980)



Equations valid only for high frequencies.

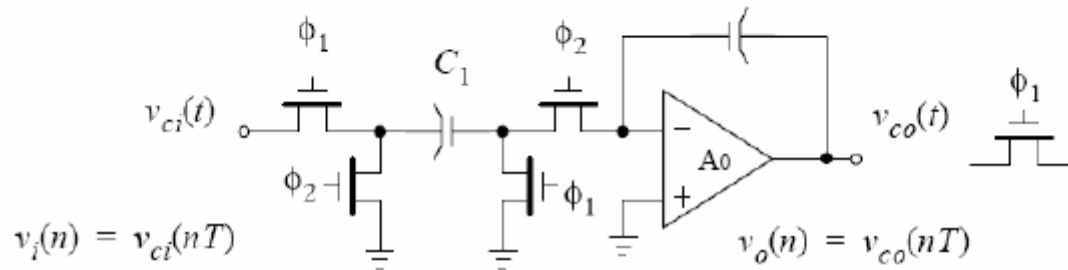
At unity-gain freq.  $\omega_i$ :  $2 \sin(\omega_i T/2) = C_1/C_2$ ,  $m(\omega_i) \sim \theta(\omega_i) \sim -1/A_0$

Usually, the magnitude error is smaller than the  $(C_1/C_2)$  error and is negligible. The phase error shifts poles/zeros horizontally, like dissipation: important!

# Finite Opamp DC Gain Effect (Cont'd)

Non-inverting integrator: similar derivation, same  $m(\omega), \theta(\omega)$ . In a biquad,  $s_p \rightarrow s_p(1 - 1/A_0)$  due to  $m(\omega)$ . The phase errors result in

$$\frac{1}{Q_p} \rightarrow \frac{1}{Q_p} + \theta_1(\omega_0) + \theta_2(\omega_0) \sim \frac{1}{Q_p} + \frac{2}{A_0 C_2}$$



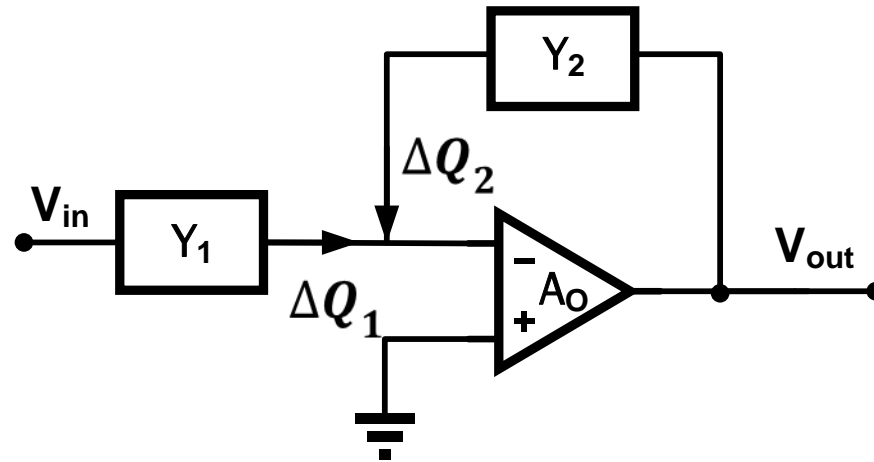
Change in peak gain:  $-20 \lg(1 + 2Q_p / A_0)$  (in dB)

can be large for  $Q_p \gg 1$  !

For  $Q_p = 15, A_0 = 1000, \Delta G \sim -0.26 \text{ dB}$

High  $Q_p \rightarrow$  use high  $A_0$  opamp! Gain-squaring integrators!

# Model for Finite - Gain Effect



$$\Delta Q_i = Y_i(V_i^+ - V_i^-)$$

is the charge flow in one clock period

$$\text{For } A_o \rightarrow \infty, \Delta Q_1 = Y_1 V_{in} = -\Delta Q_2 = -Y_2 V_o$$

$$\text{so } H(z) = V_o/V_{in} = -Y_1/Y_2$$

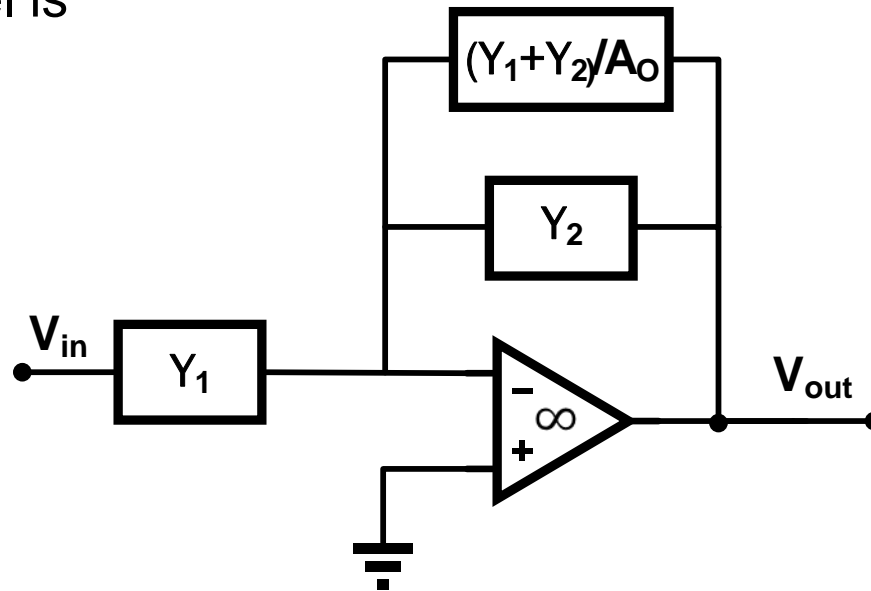
# Model for Finite - Gain Effect (Cont'd)

For finite  $A_O$ ,

$$\Delta Q_1 = Y_1(V_{in} + V_O/A_O) = -Q_2 = -Y_2(V_O + V_O/A_O)$$

$$H(z) = -Y_1/(Y_2 + Y_1/A_O + Y_2/A_O)$$

so the model is





# Correlated Level Shifting

R. Gregoire, JSSC, 2008

- Additional capacitor ( $C_{cls}$ ) at the output of the amplifier
- Working principle (integrator):

a) **Sampling phase;**

b) **Estimating phase:** coarse integration and charging of  $C_{cls}$

- Voltage left in  $C_1$ :

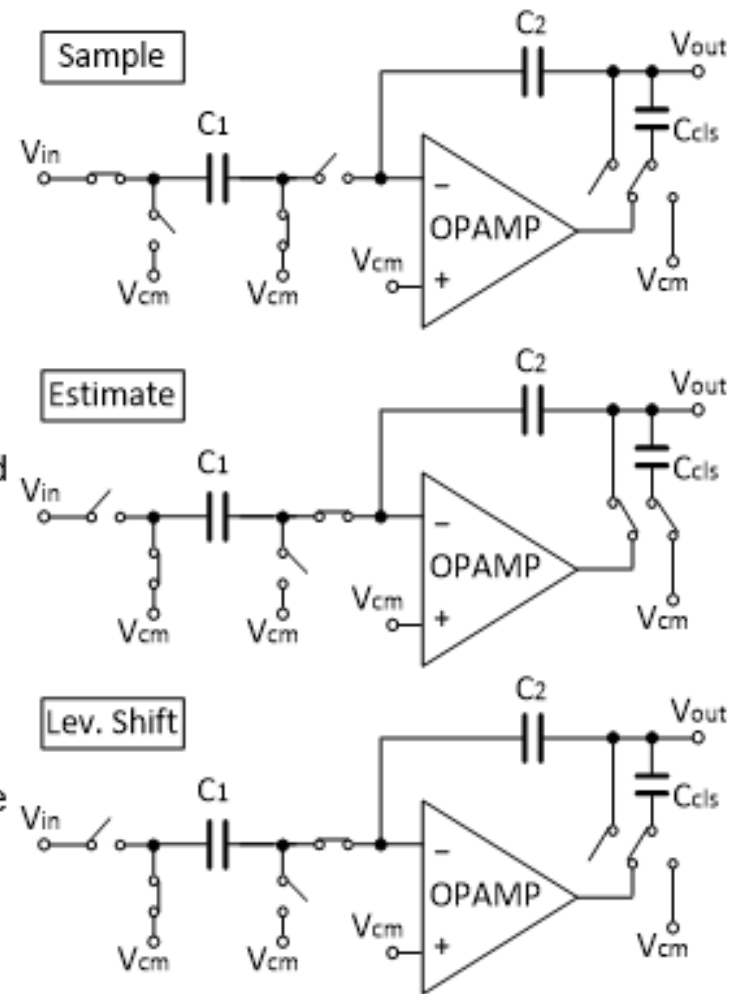
$$V_{c1(est)} = -\frac{V_{out(est)}}{A}$$

c) **Level shifting phase:** connecting  $C_{cls}$  inside the loop, and performing fine integration

- Voltage left in  $C_1$ :

$$V_{c1(ls)} = -\frac{V_{out(ls)} - V_{out(est)}}{A}$$

- For two-stage compensated opamp, the compensate cap should be included in the CLS loop.
- Ideally, CLS can boost the DC gain from  $A$  to  $A^2$ .



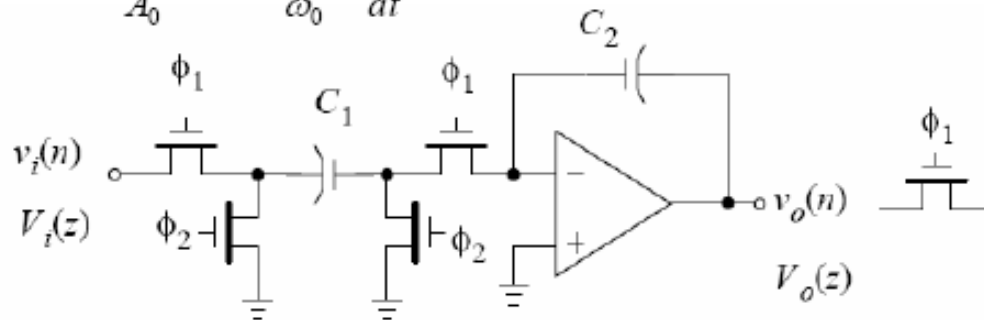
# Finite Opamp Bandwidth Effect

One-pole opamp model:

$$A_v(s) = \frac{-\omega_0}{s - s_1} \approx \frac{-1}{s/\omega_0 + 1/A_0} = \frac{V_o(s)}{V(s)} \quad \bullet \omega_0 \text{ UNITY GAIN FREQUENCY}$$

Or, in time domain,

$$\frac{1}{A_0} v_o(t) + \frac{1}{\omega_0} \frac{dv_o(t)}{dt} = -v(t)$$



Combine with KVL, charge conservation. Finding and sampling  $v_o(t)$ , calculating  $V_o(z)/V_i(z)$ , and setting  $z = e^{j\omega T}$ , for an inverting integrator results in

$$m(\omega) = -e^{-k_1} (1 - k \cos \omega T)$$

$$\theta(\omega) = -e^{-k_1} k \sin \omega T$$

Where  $k = C_2 / (C_1 + C_2)$  is the feedback factor.  $k_1 = k\omega_0 T / 2 = k\pi f_0 / f_c$  should be  $\gg 1$ .

Time constant:  $\tau = 1/(k\omega_0)$  should be  $\ll T/2$ .

# Finite Opamp Bandwidth Effect (Cont'd)

---

Since  $k_1 = k\pi\omega_0 / \omega_c$ , for  $k \sim 1$ , if  $\omega_0 \geq 5\omega_c$  then  $k_1 \geq 15$  and

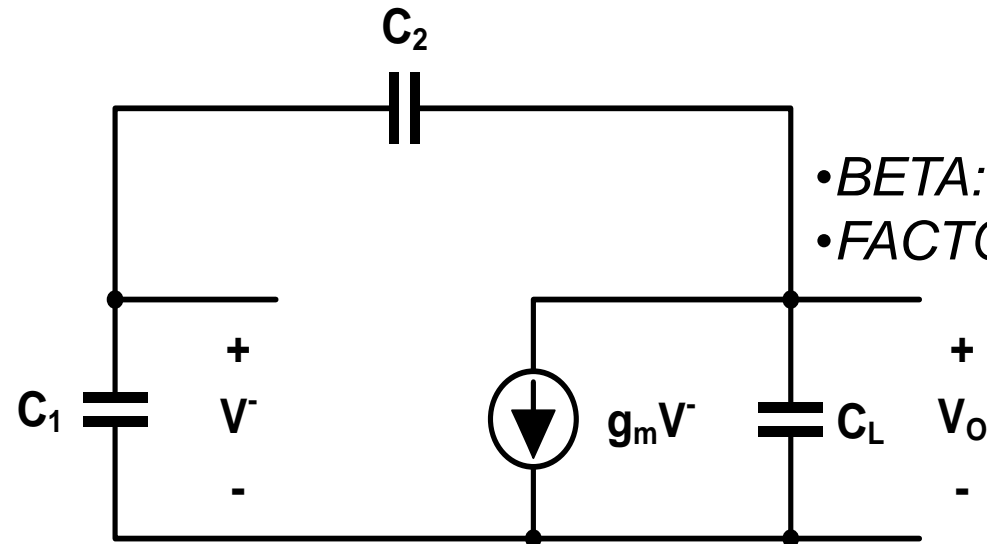
$e^{-k_1} < 3 \cdot 10^{-7}$ , so both  $m$  and  $\theta$  are negligible. Hence, for  $C_1 \ll C_2$ , use

$\omega_0 \geq 5\omega_c$  .      • *CLOCK FR. > 20. NYQUIST FREQU. FOR AAF.*

For  $k < 1$ , even higher  $\omega_0$  may be needed. Due to the exponential behavior, the error increases rapidly if  $\omega_0$  is too small!

The derivation assumes  $v_{in}(t)$  is constant. If several stages settle simultaneously, or if there is a continuous-time loop of opamp and coupling C's, then computer analysis (SWITCAP, Fang/Tsividis) is needed.

# Time Constant of OTA-SC Integrator



• *BETA: FEEDBACK FACTOR*

$$\bullet V_1 = \beta V_o \quad \bullet \beta \triangleq \frac{C_2}{C_1 + C_2}$$

$$\bullet V_1 = \frac{-\beta g_m V^-}{s(C_L + C_S)}$$

• *At pole  $S_p$ ,  $V_1 = V^-$*

$$\bullet S_p = \frac{-\beta g_m}{C_L + C_S} = \frac{-C_2 g_m}{C_L(C_1 + C_2) + C_1 C_2}$$

• *Open-loop Gain*

• *Transient term:*

$$\bullet g_m V^- + s C_L V_o + s C_S V_o = 0$$

$$\bullet e^{s_p t} = e^{-t/\tau} \quad \bullet \tau = \frac{1}{|s_p|} = \frac{C_L + C_S}{\beta g_m}$$

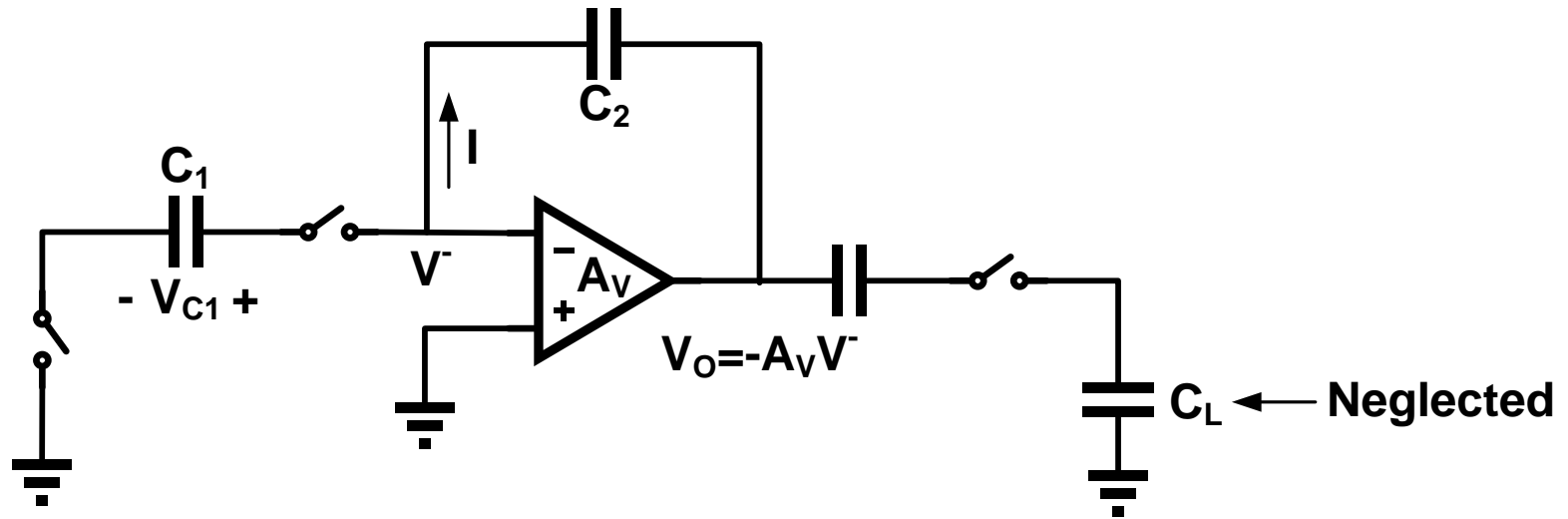
$$\bullet C_S \triangleq \frac{C_1 C_2}{C_1 + C_2}$$

• *Unity-gain conventional integrator, assuming all C is equal:*

$$\bullet V_o = \frac{-g_m V^-}{s(C_L + C_S)}$$

$$\bullet \tau = \frac{3C}{g_m}$$

# Integrator Using a Two-Stage (Buffered) Opamp (VCVS)

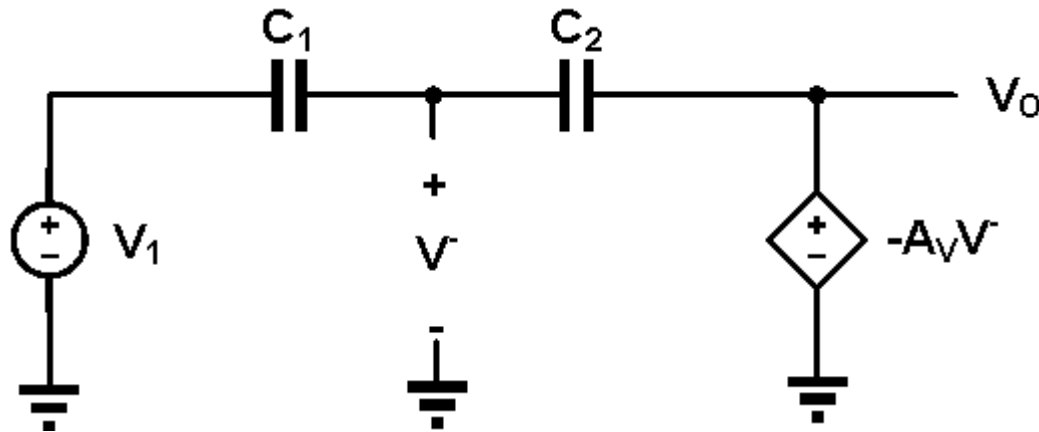


$$\bullet A_V = \frac{A_0}{s/\omega_p + 1}$$

• *Let Initial Values be  $V_{C1}=V_1$ ,  $V_{C2}=0$*

$$\bullet I = (V_1 - V_O) \frac{sC_1C_2}{C_1 + C_2} = sC_2(V^- - V_O)$$

# Integrator Using a Two-Stage (Buffer) Opamp



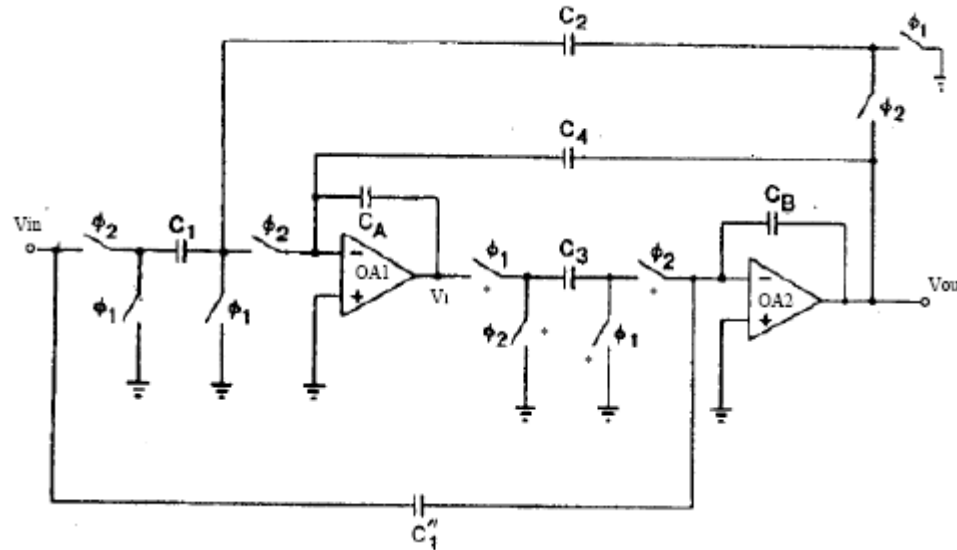
$$\bullet V_O = -A_V V^- \cong V_1 \frac{1-\beta}{s/(A_0\omega_p) + \beta + 1/A_0} \quad \bullet \beta \triangleq \frac{C_2}{C_1 + C_2}$$

$$\bullet \text{Pole at:} \quad \bullet -(\beta + 1/A_0)A_0\omega_p \cong -\beta\omega_u$$

$$\bullet \text{Time Constant:} \quad \bullet \tau \cong 1/\beta\omega_u$$

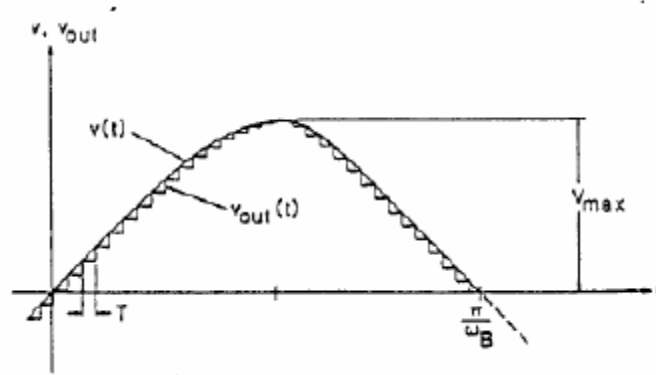
$$\bullet \text{Settling level:} \quad \bullet V_O(S)|_{s=0} = \frac{1-\beta}{\beta + 1/A_0} \cong -\frac{C_1}{C_2}$$

# High-Q Biquad

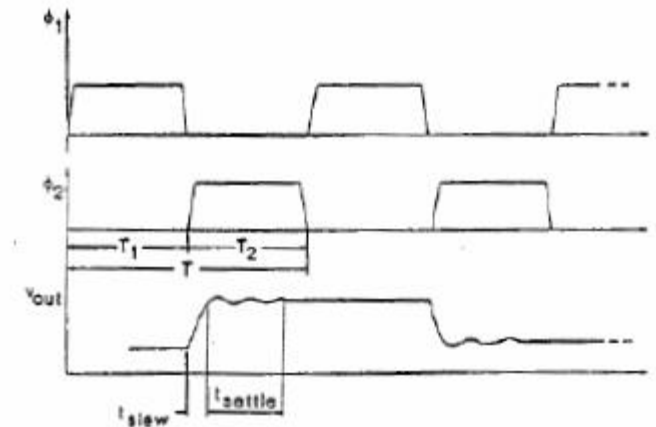


- For original phases, both opamps settle when  $\Phi_2 \rightarrow 1$ . Changing the  $\Phi_1$ , they settle separately.  $V_1$  changes twice in one cycles, but OA1 still has the same  $T/2$  time ( $T$  for the change at  $\Phi_1 \rightarrow 1$ .) to settle and to charge  $C_3$ . The transient when  $\Phi_2 \rightarrow 1$  has a full period to settle in OA1 and OA2 .

# Slew Rate Estimation (1)



Nonlinear slewing followed by linear settling:



$$t_{slew} = xT_2 \sim xT/2$$

$S_r = |(dv_{out})/(dt)|_{max}$ . For  $v(t) = V_{max} \sin \omega_B t$  where  $\omega_B$  is the maximum sine wave freq. at input, the slope  $|dv/dt|$  of the envelope  $v(t)$  is  $\leq \omega_B V_{max}$ . Then  $S_r \sim 2\omega_B V_{max} / x$ , very pessimistic estimate!



# Slew Rate Estimation (2)

---

- Much simpler estimate can be based on assuming that  $C_{in}$  is fully discharged in the slewing phase. Then the slew current can be found from
- $I_s \sim C_{in} \cdot V_{in,max} / [X \cdot T/2]$
- Less pessimistic than the previous estimate.

# Noise Considerations

---

- Clock feedthrough from switches
- External noise coupled in from substrate, power lines, etc
- Thermal and 1/f noise generated in switches and opamps

(1) Has components at  $f=0$ ,  $f_c$ , can be reduced by dummy switches, differential circuit, etc. May be signal dependent!

(2) Discussed elsewhere.

(3) Thermal noise in MOSFETs: PSD is

$$S_T = \frac{\overline{v_{nI}^2}}{\Delta f} = 4\theta R, \quad \theta = kT$$

For  $f \geq 0$ , only (one-sided distribution).

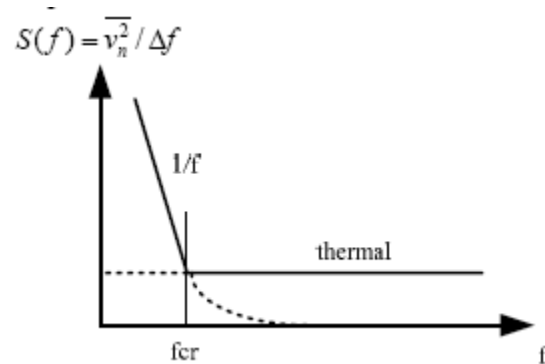
Flicker noise:

$$S_f = \frac{\overline{v_{nI}^2}}{\Delta f} = \frac{k}{C_{ox}WLf}$$

Total noise PSD:  $S = S_T + S_f$ .

# Noise Considerations (Cont'd)

- Noise spectra



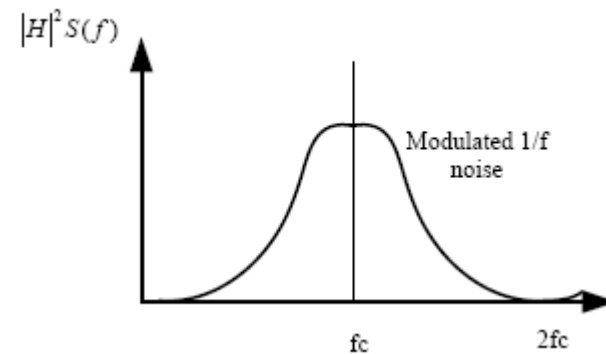
- Offset compensation (CDS—correlated double sampling); subtracts noise, T/2 second delayed.

$$H_{CDS} = 1 - e^{-j\omega T/2}$$

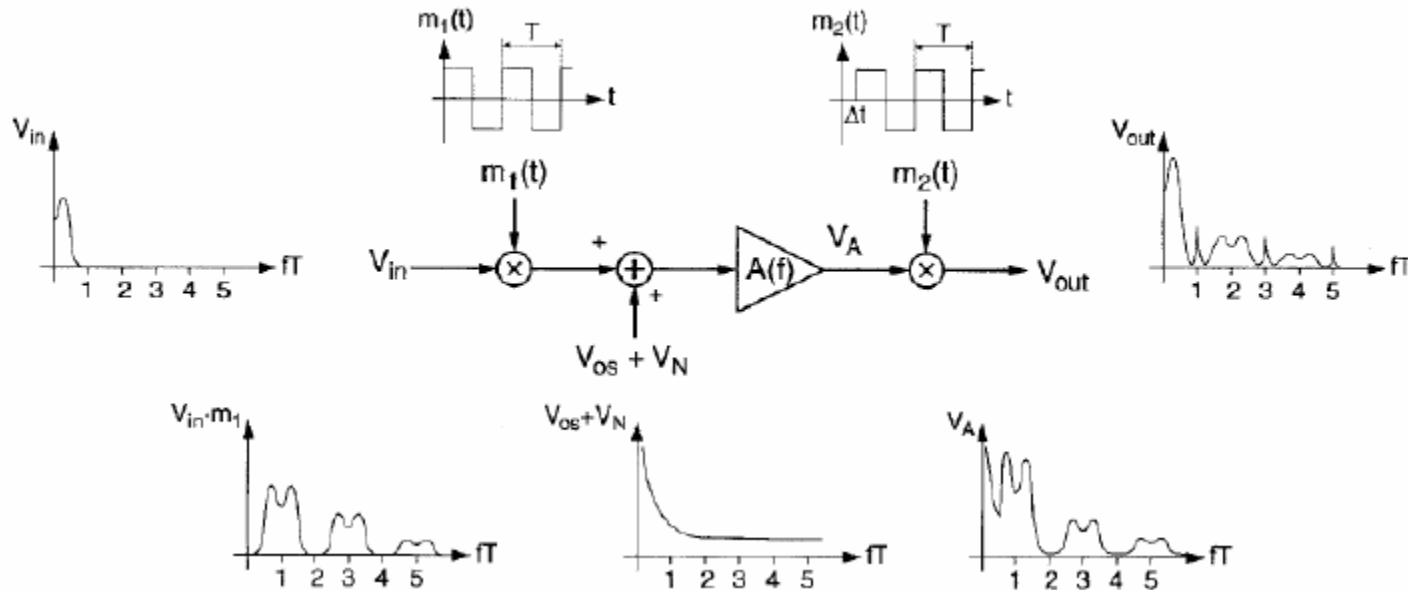
$$|H|^2 = 4 \sin^2(\omega T/4)$$

CDS:

- Pick up noise, no signal;
- Pick up noise, plus signal;
- Subtract the two.

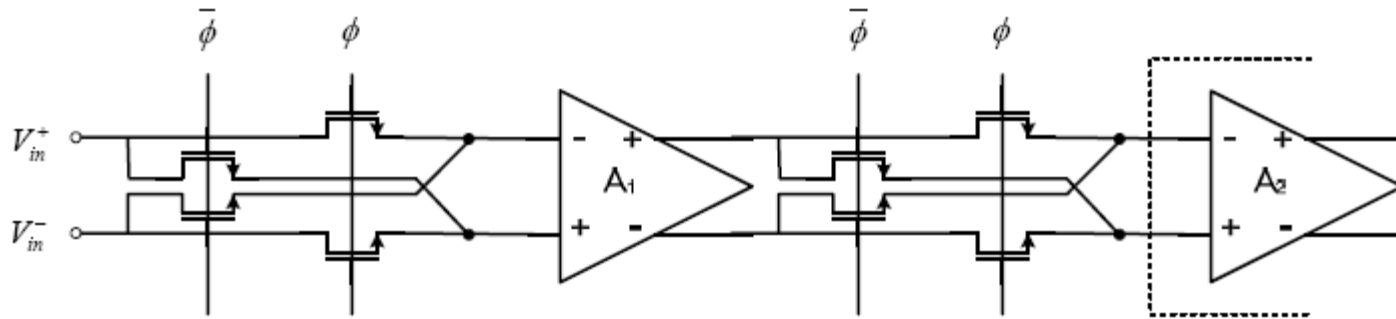


# Chopper Stabilization



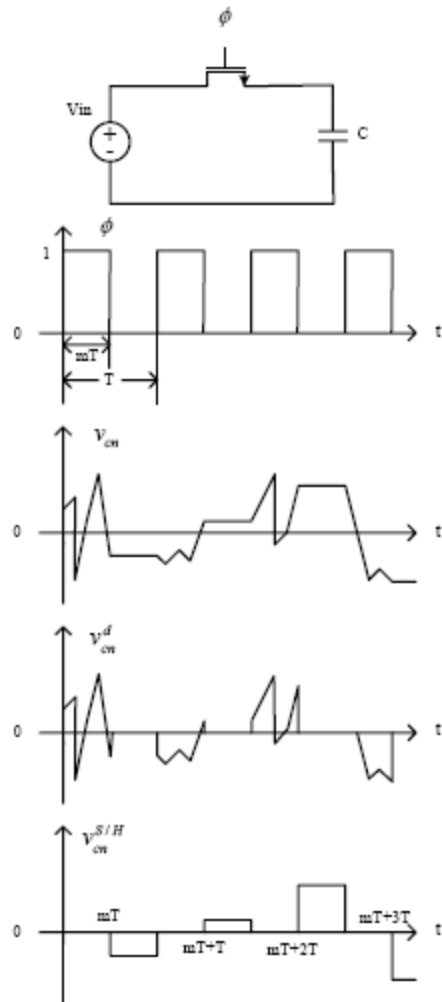
Fully differential circuits needed.

# Chopper Stabilization (Cont'd)



Differential SC amplifier using chopping.

# Noise Aliasing



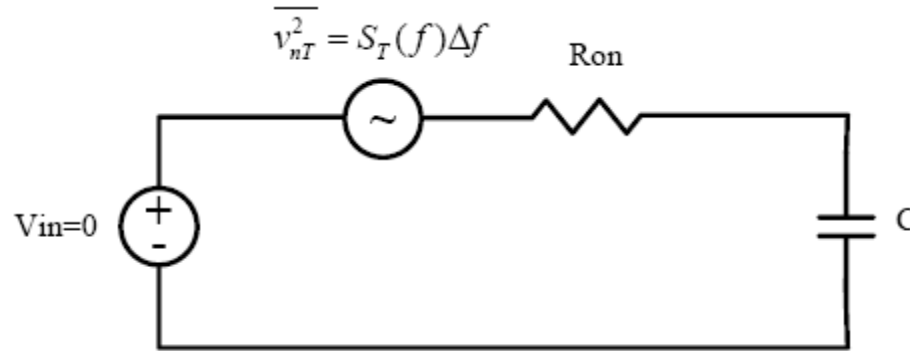
Mean-square values are the same ( $\theta / C$ ) within all windows.

Direct noise power:  $\overline{(v_{cn}^d)^2} = \frac{m\theta}{C}$

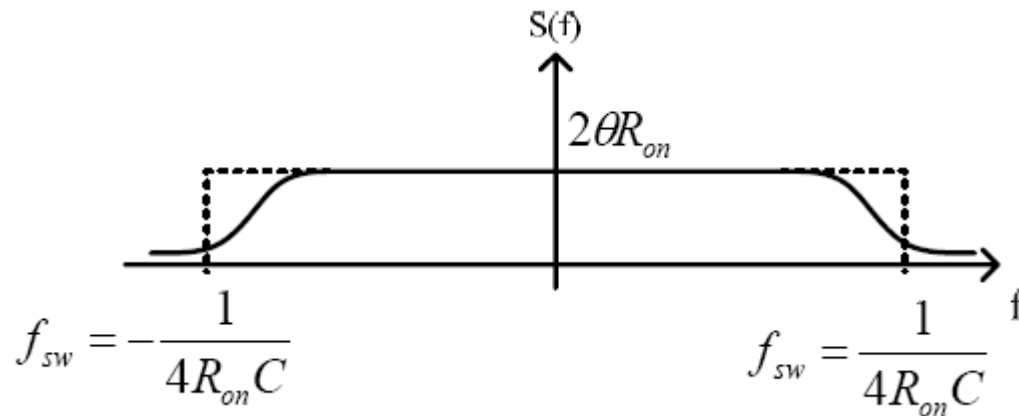
S/H PSD:  $\frac{\overline{(v_{cn}^{s/H})^2}}{\Delta f} = \left( \frac{\tau \sin f\tau\pi}{Tf\tau\pi} \right)^2 \cdot \sum_{k=-\infty}^{k=\infty} S(f - kf_c)$

S(f): RC filtered direct noise  
Most noise at dc!

# Equivalent Circuit for Direct Noise



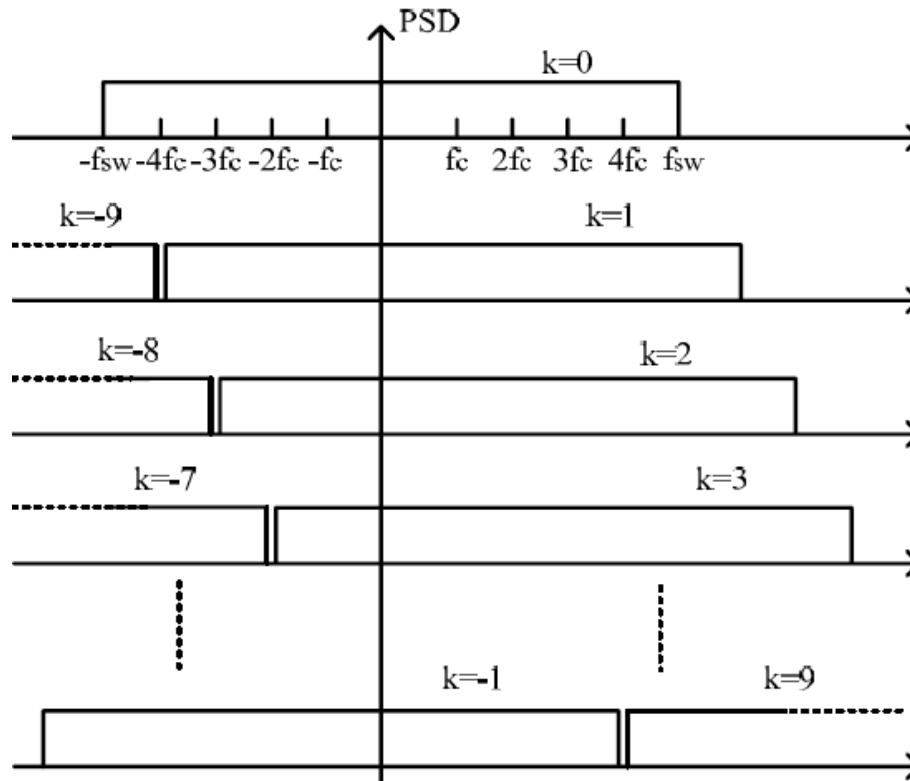
$S(f)$  for direct noise: low-pass filtered and windowed white noise.



For satisfactory settling (0.1%),  $R_{on}C \leq mT/7 = m/(7f_c)$ ,  $f_{sw} \geq 3.5f_c$ .

# Noise Aliasing

Aliasing for  $f_{sw} = 5f_c$



**$S(f)$  is magnified by  $2f_{sw} / f_c = 10$  ! The PSD is  $\theta / (Cf_c)$  after aliasing (i.e. sampling but not holding).**

**Noise power =  $kT/C$ .**

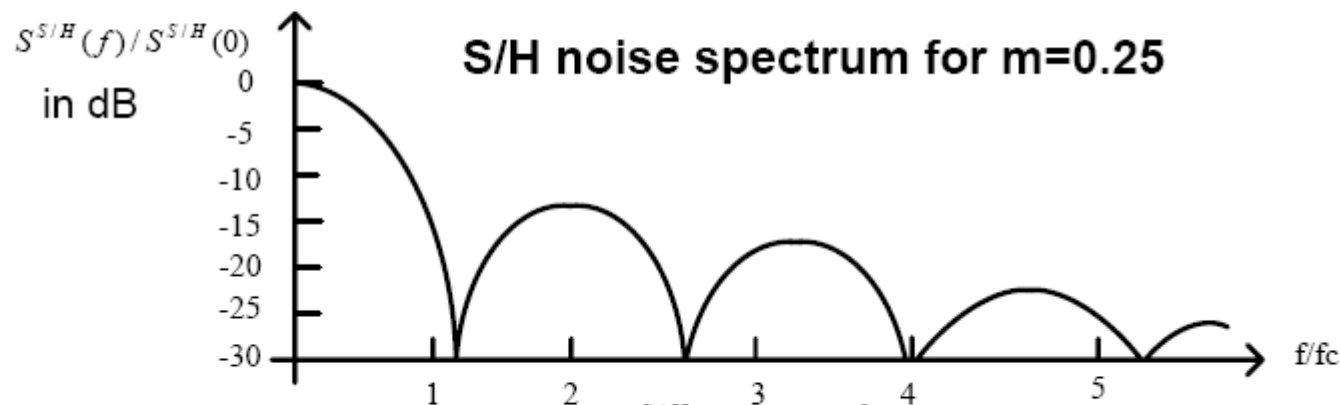


# Noise Spectra

For low frequencies ( $f \ll f_c$ )

$$S^{S/H}(f) \approx \frac{(1-m)^2 \theta}{f_c C} \quad \text{decreases with } m \uparrow$$

$$S^d(f) \approx 2m \theta R_{on} \quad \text{increases with } m \uparrow$$



Low frequency noise ratio  $r = \frac{S^{S/H}}{S^d} \approx \frac{(1-m)^2}{2m} \frac{1}{f_c R_{on} C}$

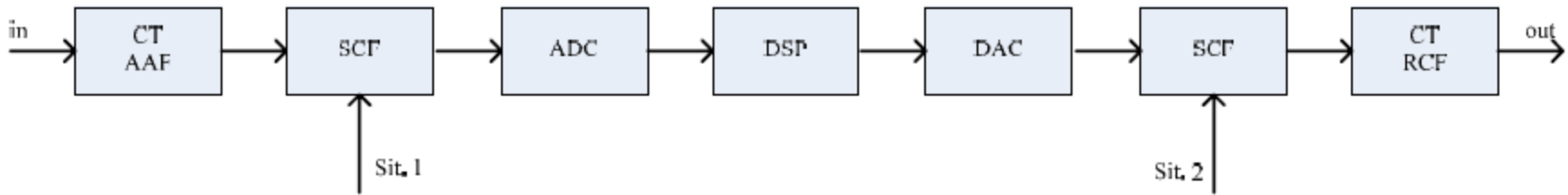
$$r \geq 3.5(1/m - 1)^2 > 3.5$$

For  $m=0.25$ ,  $r=31.5$ !

Noise generated in stage independent of  $R_{on}$ , but the noise generated in preceding stages (direct noise) gets filtered, so the  $R_{on}$  should be as large as possible!

# Switched-Capacitor Noise

Two situations; example:



Situation 1: only the sampled values of the output waveform matter; the output spectrum may be limited by the DSP, and hence  $V_{\text{RMS},n}$  reduced. Find  $V_{\text{RMS}}$  from  $\sqrt{KTC}$  charges; adjust for DSP effects.

Situation 2: the complete output waveform affects the SNR, including the S/H and direct noise components. Usually the S/H dominates. Reduced by the reconstruction filter.

# Calculation of SC Noise (Summary)

---

- In the switch-capacitor branch, when the switch is on, the capacitor charge noise is lowpass-filtered by  $R_{on}$  and  $C$ . The resulting charge noise power in  $C$  is  $kTC$ . It is a colored noise, with a noisebandwidth  $f_n = 1/(4R_{on}C)$ . The low-frequency PSD is  $4kTR_{on}$ .
- When the switch operates at a rate  $f_c \ll f_n$ , the samples of the charge noise still have the same power  $kTC$ , but spectrum is now white, with a  $PSD = 2kTC/f_c$ . For the situation when only discrete samples of the signal and noise are used, this is all that we need to know.
- For continuous-time analysis, we need to find the powers and spectra of the direct and S/H components when the switch is active. The direct noise is obtained by windowing the filtered charge noise stored in  $C$  with a periodic window containing unit pulses of length  $m/f_c$ . This operation (to a good approximation)
- simply scales the PSD, and hence the noise power, by  $m$ . The low-frequency PSD is thus  $4mkTR_{on}$ .

# Calculation of SC Noise (Summary) (Cont'd)

---

To find the PSD of the S/H noise, let the noise charge in C be sampled and- held at  $f_c$ , and then windowed by a rectangular periodic window

$$w(t)=0 \text{ for } n/f_c < t < n/f_c + m/f_c$$

$$w(t)=1 \text{ for } n/f_c + m/f_c < t < (n+1)/f_c$$

$$n=0, 1, 2, \dots$$

Note that is windowing reduces the noise power by  $(1-m)$  squared(!), since the S/H noise is not random within each period.

Usually, at low frequencies the S/H noise dominates, since it has approximately the same average power as the direct noise, but its PSD spectrum is concentrated at low frequencies. As a first estimate, its PSD can be estimated at  $2(1-m)^2 kT / (f_c C)$  for frequencies up to  $f_c/2$ .